Modeling of Rock Boundary using Walsh Domain Sequency Filtering: An Example from the German Continental Deep Drilling Program (KTB) Borehole Site

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Summary

We have developed a technique based on binary waveforms (Walsh functions) to identify rock boundaries from well log signals of KTB-HB borehole data from the depth spanning 500.024-2996.794m. We have used three sets of borehole data: e.g. resistivity, gamma ray and seismic p-wave velocity from KTB borehole. Lithologically the main structural details comprised of paragneisses, metabasites and heterogeneous series. The algorithm is based on Walsh function based sequency domain filtering which augments to resolve the bed thickness. The minimum resolvable bed thickness is a function of cut off sequency. Our analysis demonstrates that the proposed boundary detection scheme is robust for resolving and understanding structural inhomogeneity over complex metamorphic basement terrains of KTB, Germany. However, understanding the generality the method can be easily applied to large number of well log data to explore the finer structural details in sedimentary environments and modeling of stochastic signature in petroleum geophysics.

Introduction

German Continental Deep Drilling Program (KTB) (Kontinentales Tiefbohrprogramm der Bundesrepublik Deutschland) drill site is located at Windischschelbenbach, Germany (Figure 1). KTB drilling is one of the biggest geo-scientific expeditions ever undertaken in Germany which took wide attention globally for geo-scientific research. Geologically the site has been emplaced with deformed Paleozoic metamorphic rocks have been intruded by Variscan (late Paleozoic) granites (Zoback et al., 1993). In the first phase of drilling project, a pilot borehole (KTB-VB) was drilled up to 4km (completed in 1989). The second phase of the project involved drilling more depth up to 10 km in a main borehole (KTB-HB). The entire depth profile in pilot and main borehole are crosscut by several dykes and faults in additions to the major lithology changes between mainly paragneisses and metabasites and heterogeneous series (alternations between metabasites and paragneisses) (Emmermann and Lauterjung, 1997; Maiti et al., 2007, Maiti, 2009, 2010). The rocks were metamorphosed at a pressure of 6-8 kbar and a temperature of 650 °C – 700 °C. This medium grade metamorphism took place in the Lower to middle Devonian (410-380 Ma ago; Zoback et al., 1993). It is important to note that central European crust has a complex history and has been repeatedly affected by major compressional and tensional processes. It was largely formed or reshaped during the Variscan orogeny (~400-300Ma) (Emmermann and Lauterjung, 1997). The series of tectonic processes could be better understood by the crustal heterogeneities in the KTB borehole records which are well observed with varied dimensions of scale (Leonardi and Kumpel 1999).

Figure 1: Location map of the KTB boreholes

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Due to deplorable borehole conditions, borehole records are often contaminated by deceptive color noise inherently creeping into records which makes hard to detect exact rock boundary from real noisy bore hole records. Further, the interpretation depends on the subjective criteria imposed for choosing bed boundaries that may lead to different results with different conditions (Lanning and Johnson, 1983; Maiti and Tiwari, 2005; Maiti, 2009).

In such an important area of tectonically complex metamorphic terrain, exact rock boundary detections from well log records are useful for understanding of inhomogeneity at KTB site. In recent past, it has been shown that Walsh domain analysis are more appropriate and superior for bore hole records than conventional Fourier domain (sinusoidal) based analysis because of several reasons. Firstly, borehole records are naturally represent discontinuous waveform at rock boundary to tune with binary wave forms like Walsh functions. Secondly, Walsh sequency domain filtering are simple and more fast compare to Fourier frequency domain filtering to extract natural rock boundary from borehole records. During the recent years Walsh functions have been applied to various geophysical applications such as using magnetic data (Gubbins et al., 1971; Shaw et al., 2007), identifying bed boundaries from well logs (Lanning and Johnson, 1983; Maiti and Tiwari, 2005; Maiti, 2009), gravity anomaly interpretation (Keating, 1992; Shaw et al., 1998), and resistivity mapping (Pal, 1991) and in global geophysics (Tiwari 1987a, b; Negi and Tiwari 1990; Negi et al., 1993).

In this paper, we have developed a skilled algorithm based on Walsh transform technique and explored to KTB-HB records of depth interval 500.024-2996.794m which has not been analyzed so far using the proposed means.

Theory and/or Method

Walsh transforms

Walsh functions are set of orthogonal functions, form an ordered set of rectangular waveforms taking only two values either +1 or -1 within a range between 0 and 1 as shown in (Figure 2).

Figure 2: The first 8 members of the set of Walsh function. Each function takes only the values +1 and -1 with abrupt transitions between two (Beauchamp, 1975).

The mathematical complete definitions of the Walsh functions can be found in detail elsewhere (Beauchamp 1975). We give here very brief expressions to meet self sufficiency of our paper. Analogous to Fourier transform, Walsh transforms can be defined (Beauchamp 1975) as any function discretised evenly in the interval [0 1] in N number, as

\[ x_t = \sum_{j=0}^{N-1} a_j Wal_j(x_t) \quad \text{for } t = 0 \]  

Where \( N \) is a power of two and integrable in Lebesgue sense can be defined as sum of N Walsh functions so that

\[ x_t = N \sum_{j=0}^{N-1} a_j Wal_j(x_t) = N \delta f_t \]  

Because Walsh functions obey orthogonal property, so we can write,

\[ \sum_{t=0}^{N-1} Wal_i(x_t)Wal_j(x_t) = N \delta_{ij} \]  

So if we multiply both sides of above eq. (2) by \( Wal_i(x_t) \) and sum over \( t \) to find that

\[ a_j = \frac{1}{N} \sum_{t=0}^{N-1} x_t Wal_j(x_t) \]  

Points \( a_j = \frac{1}{N} \sum_{t=0}^{N-1} x_t Wal_j(x_t) \) are the Discrete Walsh transform (DWT) of the set of values \( x_1, \ldots, x_N \). The direct and inverse transforms are identical (except for the normalization by \( 1/N \)) because of the symmetry of Hadamard matrix. DWT is mathematically correct, but it is computationally slow when a large number of data points are analyzed since there \( N^2 \)
operations (additions and subtractions) are required. But Fast Walsh transform (FWT) can be reduced to 
\[ N \log_2 N \] (additions and subtractions) instead of \( N^2 \) operations (additions and subtractions) in DWT (Beauchamp 1975). Similar to frequency, sequency is defined as the average number of zero crossing per unit interval/ generalized frequency (Harmuth, 1968). The Walsh transform technique is more appropriate having no Gibbs phenomena to model the discontinuous square wave rather than continuous sinusoidal wave (Bath 1974). To demonstrate, we compare power spectrum of 16Hz sine wave of Fourier and Walsh results which show that there is no side lobes of energy apart from spike at 16Hz in frequency but significant side lobes in Walsh sequency domain whereas the picture is reverse when analyzing 16Zps (zero crossing per second) square wave (Figure 3&4). The sequency domain filtering is done basically to separate desired spectral properties (signal) from undesired ones (noise). Unlike the frequency domain filtering, Walsh sequency domain filtering is done by setting filter coefficients either 0 or 1 (Harmuth, 1968). Adopting this concept, all types of filter, low pass, band pass, high pass and notch sequency filters can be designed. For low pass filtering operations done here in Walsh domain, all sequencies above certain threshold sequency (we call cut-off sequency) are multiplied (or replaced) by 0, and others are remains unchanged. The final results are inverse Walsh transform of the low pass filtered records, which is stepped version of the original records. It is very interesting to observe with our synthetic experiments that Walsh low pass versions can able to extract binary model from original random signals (Figure 3) even with random signal corrupted with noise (Figure 4).

**Boundary detection algorithm**

Boundary detection algorithm is applied on Walsh low pass filtered data. However, we have described (Maiti and Tiwari, 2005; Maiti, 2009) that unlike Lanning and Johnson (1983) the depth, beginning of the records may not always be a boundary instead we call it mean boundary value. The procedure is to move to next step on one of the low passed records and computes the absolute magnitude difference between present and the mean of previous value on all previous steps since last boundary was detected. This absolute difference is computed for all borehole records simultaneously (here three). Each is multiplied by weighting coefficients and results are summed as to form “Walsh Pick” value. It is important to note that “Walsh Pick” is non-dimensional number which measures the amount of change taking place simultaneously on all low pass records. “Walsh Pick” is compared with another non-dimensional number called “Check” which is assigned by the interpreter. It is conditioned that if “Walsh Pick” is greater than or equal to the “Check”, the depth at the beginning of the step is detected as a boundary; otherwise a new boundary has not been crossed and the procedure is repeated.

**Examples**

The boundary detection method described above is applied to identify the rock boundary from the KTB main bore hole (KTB-HB) records of depth interval 500.024-2996.794m. The main bore hole records are sampled at 0.1524m (6 inch) interval. The total depth of the main hole is 9001m.
In the present study three types of records resistivity, gamma ray intensity and seismic p-wave velocity are used to detect rock boundary. For the purpose of filtering and comparison each set consist of three data sets from a common depth. Original borehole records (Figure 5) & Walsh low passed version (Figure 6) of resistivity, gamma ray and seismic p-wave velocity shows clear picture that how Walsh low passed version extracting natural changes of rock type over KTB main hole from depth 500.024-2996.794 m. The step-width here defines the maximum resolvable layer thickness is constant (39.066m) for entire well log records considered.

The thickness of layer less than the step width can be detected but not to be resolved using this method (Maiti and Tiwari, 2005; Maiti, 2009). The step width is a function of cut-off sequency in Walsh low pass filtering operation and therefore adjustable; lower the cut-off sequency, the wider the step-width of the Walsh low passed data. (Lanning and Johnson, 1983, Maiti and Tiwari 2005; Maiti, 2009). Thus step width = total band width/width of the low pass band, i.e. step width is constant, if cut-off sequency is varied from $2^{N-1} + 1$ to $2^N$ where $N$ is number of data points used in Walsh low pass filtering. Step width is constant over binary band varying cut-off sequency from 64(Zpm) to 33(Zpm) where 16384 data points are used (Figure 6). By changing cut-off sequency, one can set step width in order to resolve minimum thickness of the litho logic layer with specific to the problem in hand. However there should be a proper trade off between the selection of the cut-off sequency and noise present in the data. In the present case, cut-off sequency of well records is chosen in such a way that minimum resolvable layer thickness formed step width of 39.066m in all Walsh low passed versions as mentioned earlier.

Weighting coefficients are usually assigned as inverse of total number of log records used i.e., 0.33 for three bore hole records. The idea behind is to give equal weightage to each log responses. However, one can have any value between 0 & 1 according to variance of log signal in hand against the particular depth/space section. Accordingly weighting coefficients, 0.2 for resistivity, 0.75 for gamma ray and 0.5 for seismic p-wave velocity produced most stable and consistent results for present data analysis. Although we do not have any a priori information about “Check” value, we assigned a range of “Check” values from 0.05 to 0.08 for present analysis on trial and error basis to produce stable and satisfactory results. The boundary detection for real bore hole log example is presented in figure 7 where the check value is 0.05 and cut-off sequency is 40 Zpm (Zero crossing per meter).

Figure 5: KTB main hole well log responses, resistivity (ohm-m), gamma ray intensity (A.P.I.) and seismic p-wave velocity (km/s) from 500.024-2996.794 m are normalized between [0.0 1.0], [0.5 1.0] and [0.8 1.0] respectively.

Figure 6: Walsh low pass version of the density, resistivity and P-wave velocity data from depth 500.024-2996.794 m are plotted, scale for resistivity, gamma ray intensity and P-wave velocity are [0.3 1.0], [0.6 1.0] and [0.95 1.0] respectively.

Figure 7: Walsh boundary picking over the real example from depth 500.024-2996.794 m where the cut off sequency is 40(Zpm) and scale for resistivity, gamma ray and p-wave velocity are [0.5 1.0], [0.6 1.0] and [0.95 1.0] respectively.
It can be clearly observed that some boundary identified in one low passed version (Figure 6) but not at the other low passed version of the same records (Figure 7). In this regard it should be noted that Walsh method is able to detect rock boundary correctly up to one half of the step width because Walsh transform is a real number operation which do not carry any phase information (Lanning and Johnson 1983).

Following Lanning and Johnson (1983), we define the formula to correct the depth of boundary as

$$B_i = W_i \pm \Delta sw/2; \quad \text{............... (4)}$$

Where $B_i$ is true location of $i^{th}$ the boundary, $W_i$ is the depth detected by Walsh method. $\Delta sw$ is the step width of the low passed version of well log signal. Relative energy build up is shifted in second low passing operations depending on the true position of the rock boundary. How energy build up being shifted in each Walsh low passing operation for boundary detection procedure is demonstrated in figure 7.

So to get true position of the boundary, two sets of boundary can be averaged as final.

$$B_i = W'_i \pm \Delta sw/2; \quad \text{............... (5)}$$

where $W'_i$ is the final $i^{th}$ boundary by Walsh method.

The each process leads to increase of resolution by a factor of two.

Finally using the above method, the identified boundaries are plotted over the three sets of original log data (Figure 8). Figure 7 shows clear picture of rock boundary detection by Walsh technique which is well replicated by abrupt and significant change in all log response matching with signal characteristics. It is also observed that major bed boundary detected at depth 1182.776m and 1416.863m which is very closely matched with published litho-section of KTB site (Figure 8), in addition, some other bed boundary at depth 734.111m, 987.500m, 1923.500m and 2021.586m also detected and presented by dotted line (left) on original log response (Figure 8). From the same figure we can infer that the depth range 1182.776-1416.863m shows very low gamma ray clearly demarcate the bed boundary of metabasites unit from paragneisses unit which also supports the existent geological sections over KTB (see Maiti et al., 2007). In additions, from figure 8, we can infer that the high gamma ray and low resistivity response in depth interval 1923.500- 2021.586m indicates the presence of heterogeneous series(alternations between metabasite and paragneisses) which was not reported in previous studies(see figure 2 of Maiti et al., 2007 litho-section(right)). It is also important to note that high concentrations of potassium (K), thorium(Th), and uranium(U) in crystalline successions reveal the presence of acidic rocks such as paragneisses, whereas basic compositions are reflected by a scarcity of radio nuclides (Leonardi and Kumpel, 1999) which provides better guidance to interpret log response recorded against a metamorphic successions of KTB super deep bore hole. Depending on geophysical parameter/response with geological information, geoscientists grossly classified major litho-facies unit over KTB site but using this proposed method, in addition to the existing main boundary, we explore and report some other depth of lithologic boundary associated with small scale change of log response pertaining to existent finer structure, which were not reported at previous studies at KTB-HB which exemplify the potentiality of this method to detect finer structure which is present at KTB site.
Conclusions

A litho log boundary detection technique based on the Walsh function theory is proposed and applied to the KTB-HB data of depth interval 500.024-2996.794m. The proposed method is simple, fast and more effective, compare to other traditional method. In particular, the method provides an effective means of matching natural well log boundary. The proposed method is able to clearly identify known litho logic unit, in addition to some finer structures, which were not detected in earlier investigation. Geological information confirms the presence of these finer structures. This method provides satisfactory results even in the presence of noise in the signal at the KTB site. The method is effective to handle large amount of bore hole record, so that more fine structure/structures can be resolved at the cost of comparatively less computing time. The proposed method finds implication for resolving litholog boundaries and understanding the crustal inhomogeneity over KTB-HB super deep well site.

References


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