Pre Stack Migration Aperture – An Overview

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Summary

Primary reflections and diffractions are the main target of seismic migration. Migration is also needed to improve lateral resolution and preserve amplitudes even for horizontal reflectors that do not exhibit dip. For proper pre stack migration of seismic data, appropriate migration aperture needs to be estimated that can take care of structural dips and fault definitions. Considering diffraction limbs of a point difractor, migration aperture is often estimated as large as possible. However, this has implications on data acquisition and processing. An attempt is made to present the various concepts involving pre-stack migration aperture.

Introduction

For convenience, we choose a homogeneous medium of velocity v=2000m/s and use distance (tv/2) coordinate instead of travel time. Let us recapitulate the post stack case where the seismic reflections are shown at their source-receiver coincident position i.e. CMP location on the surface and extend the concepts to pre-stack case.

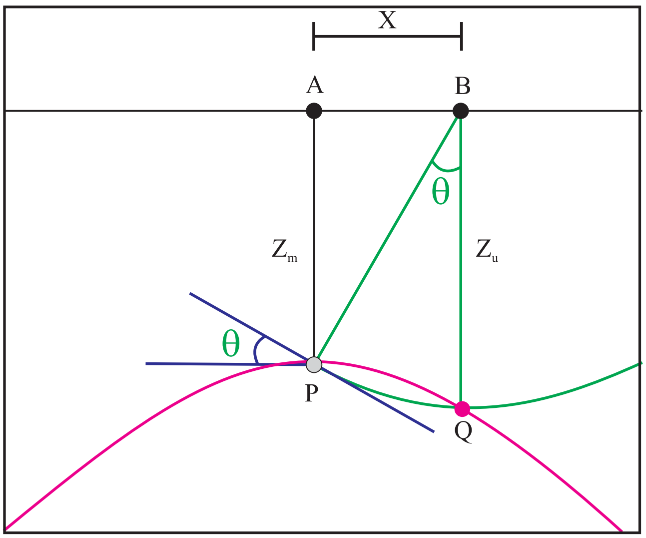

Fig.1. Locus of reflections from Point difractor for all dips in hyperbola

Consider a point A on the surface and a point difractor P below it, at a depth Z_m as shown in figure 1. The point difractor is a reflecting point that is common to all dipping reflectors passing through P. A plane reflector at an angle θ at P gives rise to a reflection that is received at B on the surface at a distance x from A. When looking from B, it appears as if the reflection is coming from a point Q on a horizontal reflector at a depth Z_u (BQ=BP). Even though the reflection is generated from a point P on a dipping reflector below surface position A (migrated position), it is recorded at Q (un migrated position) below the surface position B on seismic section. The suffix 'm' indicates migrated and the suffix 'u' indicates un-migrated. The recorded event at un-migrated position Q has to be moved by a horizontal distance x and by a vertical distance (Z_u - Z_m) to P where it belongs (i.e. its migrated position). The horizontal distance x between the surface positions A and B is the migration distance for event P at a depth Z_m. The relations between x, Z_m, θ. and Z_u are straight forward and are given below. Of these four variables only two are independent.

$$Z_u = Z_m / \cos\theta = \sqrt{Z_m^{\,2} + x^2} \qquad \text{......... (1)}$$

$$x = Z_m \tan\theta = Z_u \sin\theta \qquad \text{......... (2)}$$

$$\cos\theta = Z_m / Z_u \qquad \text{......... (3)}$$

The diffraction hyperbola is the locus of all reflection events Q (x, Z_u), each from a different dip, emanated from point difractor P (x=0, Z_m) at the apex of the hyperbola. Figure 2 shows the normalised hyperbola for a point difractor at x=0 and Z_m=1Unit (to get x and Z_u, multiply the values read from the graph with Z_m).

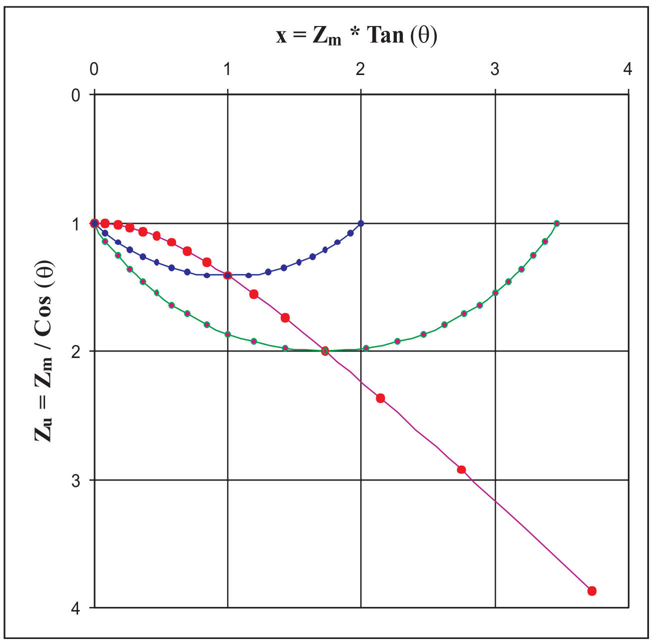

Fig.2. Diffraction hyperbola for Z_u=1 unit

The points (red dots) shown on the hyperbola are at uniform intervals of reflector angles θ from 0^0 to 75^0 with an increment of 5^0. As the reflector angle tends to 90^0, the migration distance x tends to infinity. The two arrows show the points on the diffraction hyperbola corresponding to 45 and 60 degree reflectors.

Migration is done to reposition the reflection events to their migrated positions. In migration as diffraction summation method, the reflection events at their recorded (un migrated) positions $Q(x,Z_u)$ are summed along the diffraction hyperbola to get the output reflection point P at its migrated position. In migration as semicircle superposition method, each reflection event at its un-migrated position Q is mapped to all possible output positions to which the travel time is constant (semicircle) about the recorded position B. The methods of hyperbola summation and semicircle superposition are equivalent migration methods.

In diffraction summation method, theoretically, it requires integrating all the amplitudes along the hyperbola extending up to infinity and placing the amplitude at the apex of the hyperbola. Practically it is not possible as the events are only recorded up to a maximum record time and the equivalent Z_{umax} decides the maximum migration distance

$$X_{max} = Z_m \text{ Tan } (\text{Inv Cos}(Z_m/Z_{umax})).$$

Let us consider the amplitude of the reflection event at Q (for a given dip of reflector at P) relative to the amplitude at P (for zero dip).

Amplitude reduction at Q relative to Amplitude at P
due spherical divergence = Amplitude at P * Z_m/Z_u
due to directivity = Amplitude at P * Cos θ
Total Amplitude reduction = Amplitude at P * (Z_m/Z_u)**2
at Q relative to P = Amplitude at P * $\text{Cos}^2\,\theta$

Integrating $\text{Cos}^2\theta$ from 0 to θ degrees gives the contribution to the amplitude due to limited migration distance represented by θ. The percentage amplitude as a function of migration distance (represented as angle) is shown in figure 3 to understand the effect of summing over a limited migration distance. It is seen that summing the amplitudes up to a migration distance $x = 1.732 * Z_m$, equivalent to 60 degrees, contributes 95% to the integrated amplitude. At this distance, the amplitude is ¼ of the amplitude at the apex. Claerbout [1], describes the maximum width of the hyperbola as the one containing three quarters of energy in the hyperbola that is equivalent to about 87% of the amplitude or approximately 50^0 dip and migration distance x equal to 1.2 times the depth of the reflector Z_m.

Migration distance for different angles is given below.

Angle at ¾ Amplitude = 30.0^0
and migration distance $x = 0.577 * Z_m$
Angle at ½ Amplitude = 45.0^0
and migration distance $x = 1.000 * Z_m$
Angle at ¼ Amplitude = 60.0^0
and migration distance $x = 1.732 * Z_m$
Angle at $^1/_{16}$ Amplitude = 75.5^0
and migration distance $x = 3.873 * Z_m$

For collapsing diffraction events from a point difractor, considering migration distances beyond a limit only adds to cost without further improvement.

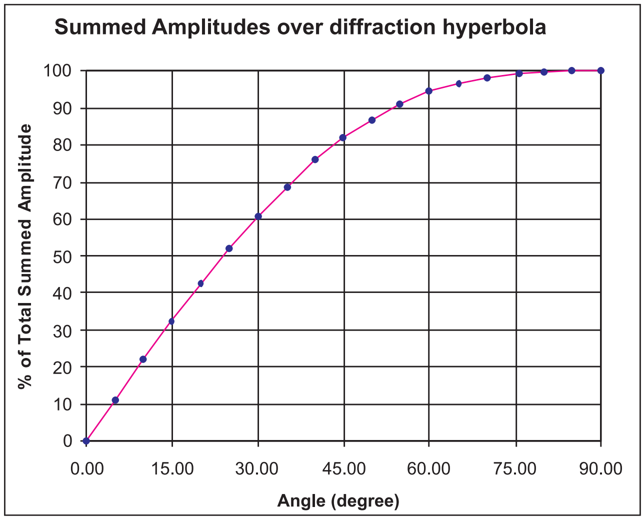

Fig.3. Percentage amplitude along diffraction hyperbola

Migration distance has an impact on the acquisition and processing costs. Therefore, it is necessary to judiciously choose the migration distance. Even for a plane reflector with zero dip, the reflection energy comes from an area below the CMP position defined by the Fresnel zone. Considering the Fresnel zone radius may resolve the lateral extents of the zero dip reflectors, but it is not adequate to collapse the diffraction events and preserve the reflection amplitudes.

The migration distance for the anticline shown in figure 4a is computed based on the structural dips only. If the same structure is faulted (Fig 4b), the migration distance needs to be computed for both structural dips and for diffractions from fault planes.

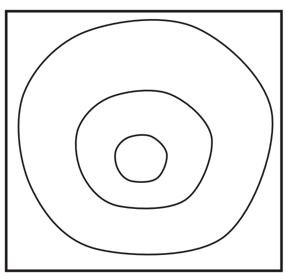

Fig.4a. Migrating gentle dips need lesser migration distance

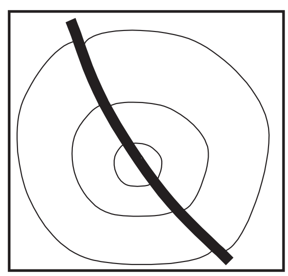

Fig.4b. Migrating fault plane needs larger migration distance

T(x)**2 = t(0)**2 + 4x**2/v**2

Z_u(x)=z_m**2+x**2

Minimum migration aperture based on Fresnel zone

Considering a point P on a dipping reflector at a depth Z_m, the migration distance can be estimated using the reflection geometry for any given source and receiver pair.

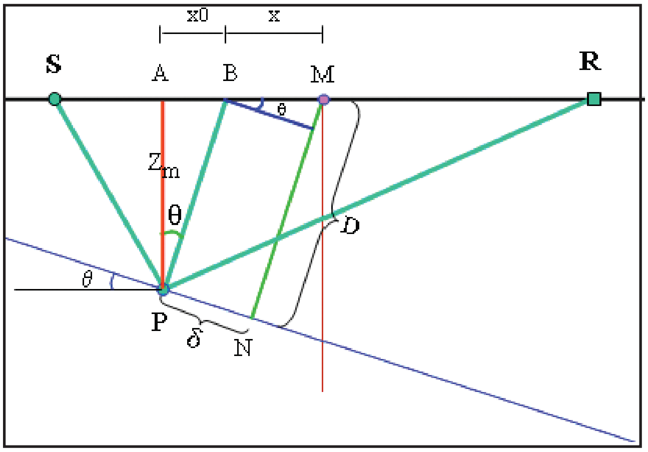

Fig.5. Reflection from a dipping reflector into a Source-Receiver pair

Figure 5 shows the reflection geometry for a point P on the dipping reflector θ. The ray emanating from the source S is reflected at P and received at receiver R. The reflecting point P is directly below A on the surface at a depth of Z_m. The reflection event recorded from P is placed below the source receiver midpoint M. The perpendicular to the reflector at P cuts the surface at B and not at the source receiver midpoint M. MN is the perpendicular from midpoint M to the reflector. Levin [2] showed that the distance NP=δ is equal to

$$\delta = h^2 \sin 2\theta \;/\; 2D \qquad \text{......... (4)}$$

Where h is half of the source receiver distance and D is the length MN.

The migration distance from midpoint M to A above the reflection point on the surface is given by x = x0 + x1**.**

Here x0 is the zero offset migration distance i.e. $Z_m \tan\theta$

and $x1 = \delta / \cos\theta$

$$\delta = h^2 \sin 2\theta \;/\; 2D = x1 \cos\theta \qquad \text{......... (5)}$$

$$D \text{ can be written as } (PB + x1 \sin\theta) \qquad \text{......... (6)}$$

$$D = (Z_m + x1 \sin\theta \cos\theta) / \cos\theta \qquad \text{......... (7)}$$

Substituting D in equation 5 and rearranging the terms one gets

$$x1^2 \sin 2\theta + 2 Z_m x1 - h^2 \sin 2\theta = 0 \qquad \text{......... (8)}$$

$$\text{or } x1 = (-Z_m + \mathrm{Sqrt}(Z^2_m + h^2 \sin^2 2\theta)) / \sin 2\theta \qquad \text{......... (9)}$$

Migration distance $= Z_m \tan\theta + (-Zm + \mathrm{Sqrt}(Z^2_m + h^2 \sin^2 2\theta)) / \sin 2\theta$ .... (10)

$$\text{Migration distance } x = \frac{-Z_m \cos 2\theta + \sqrt{Z^2_m + h^2 \sin^2 2\theta}}{\sin 2\theta} \qquad \text{....(11)}$$

The migration distance is the sum of zero offset term x0 (source receiver coincident) and the offset term x1. In this case, the source receiver coincident point (B perpendicular to the reflector at P) corresponding to the zero offset term is not the actual source receiver midpoint (M). However, the reflection event is positioned below M and not below B. When looking from M, it appears as if the reflection is coming from a point Q on a horizontal reflector at a depth Z_u such that the reflection travel path distance is same as that for the reflection from P (SPR=SQR). These two events are indistinguishable. Pre-stack migration resolves these events. The locus of all the reflection points with the same travel path distance (time) from a given pair of source receiver is an ellipse for homogeneous medium. The reflection energy recorded below the source-receiver midpoint is moved along the ellipse in case of pre-stack migration. The special case of zero offset is the post stack migration. The pre-stack migration ellipse is shown in figure 6.

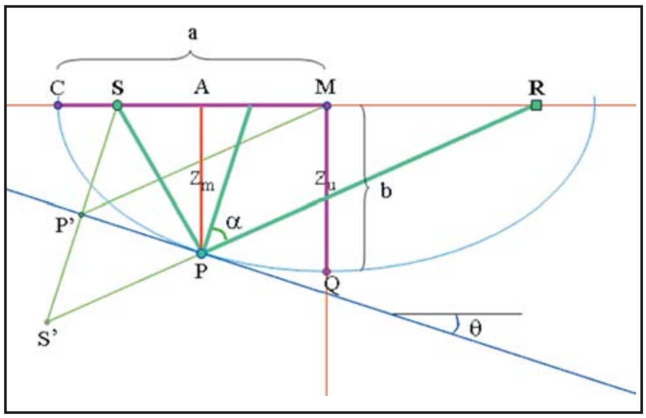

Fig.6. Geometry of Migration ellipse for pre-stack migration

The reflector with dip θ is tangential to the ellipse at point P from where the reflection is generated. The angle of reflection at P is α. The depth of the reflection point P is Z_m at the migrated position A and the depth of the reflector with zero dip is Z_u at the un-migrated position M. S' is the image of source as seen from the receiver. From the diagram Δ^{le} SS'R and Δ^{le} SP'M are isosceles and SM=MR=h. MP'=RS'/2. Therefore semi major axis of the ellipse '**a**' is equal to MP'=SPR/2=SQ and the semi minor axis '**b**' is equal to Z_u =Sqrt(SQ**2 - SM**2).

Considering the origin at M(0,0), Source at S(-h,0) and Receiver at R(h,0), the equation for the semi-minor axis b is obtained as

$$b = \frac{h}{n \cos\theta} \sqrt{\frac{1 + \sqrt{1 + n^2 \sin^2 2\theta}}{2}} \qquad \text{......... (12)}$$

here n = h/Z_m. .... (13)

$$a = \sqrt{b^2 + h^2} \qquad \text{......... (14)}$$

For a given h and θ, the relation between Z_m and Z_u=b is given by equation 12.

The point P(x, Z_m) is also a point on the ellipse defined by [**a**, **b**], i.e.

$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$$

and hence one can find the migration distance from the equation of the ellipse at z=Z_m where the tangent to the ellipse becomes the dipping reflector with dip angle θ. From the equation of the tangent to the ellipse, the following relation for migration distance is obtained.

$$x = Z_m \frac{a^2}{b^2} \text{ Tan}\theta \qquad \text{......... (15)}$$

Equation 11 and equation 15 give the same result for any Z_m and θ.

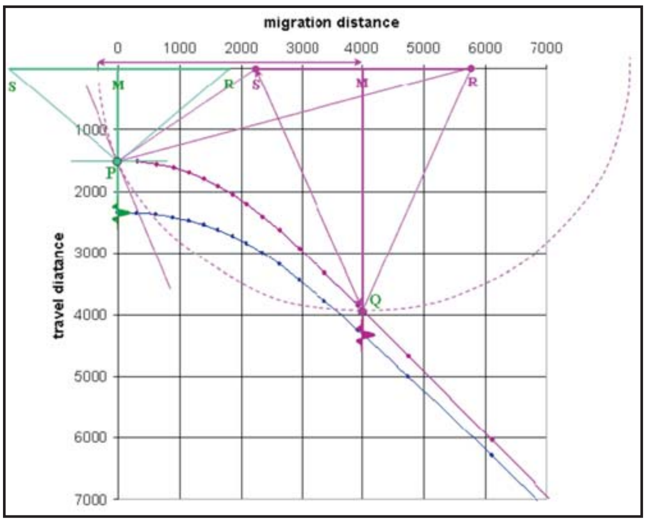

Fig.7. Diffraction hyperbola for h=1800m and Z_m =1500m.

To find the offset section from a point difractor, i.e. the diffraction hyperbola for offset case, the migrated position Z_m (P) is kept constant and migration distance x and the half travel distance from source to receiver SPR = '**a**' are computed for various dipping reflectors at P. Such a graph corresponding to half offset h=1800m and Z_m=1500m is shown in figure 7. Referring to figure 7, the pink curve shown corresponds to **b**=Z_u, while the black curve represents the half travel distance '**a**' where the events will be recorded on the offset section.

Now let us compute the migration distance for different depths Z_m at migrated positions for a constant dip of the reflectors i.e. for a series of parallel reflectors.

h=1800, reflector dip = 18 degrees						Angle of reflection β at Q (Z_u)
				migration distance		
Z_m	*n*	*b*	*a*	*Eqn. 15*	*Eqn. 11*	
900	2.00	1067	2093	1124.4	1124.4	59
1000	1.80	1165	2144	1100.4	1100.4	57
1100	1.64	1264	2199	1082.6	1082.6	55
1200	1.50	1363	2258	1070.1	1070.1	53
1300	1.38	1462	2319	1062.3	1062.3	51
1400	1.29	1563	2384	1058.5	1058.5	49
1456	**1.24**	**1619**	**2421**	**1058.0**	**1058.0**	**48**
1500	1.20	1663	2451	1058.3	1058.3	47
1600	1.13	1764	2520	1061.2	1061.2	46
1700	1.06	1865	2592	1066.7	1066.7	44
1800	1.00	1967	2666	1074.7	1074.7	42
1900	0.95	2069	2742	1084.7	1084.7	41
2000	0.90	2171	2820	1096.6	1096.6	40
2100	0.86	2273	2900	1110.2	1110.2	38
2200	0.82	2376	2981	1125.2	1125.2	37
2300	0.78	2479	3063	1141.5	1141.5	36
2400	0.75	2581	3147	1159.0	1159.0	35
2500	0.72	2684	3232	1177.5	1177.5	34
2600	0.69	2788	3318	1197.0	1197.0	33

Reflection angle for the zero dip reflectors at depths Z_u that have the same travel distance as that of the dipping reflectors at Z_m are also computed and given in the table. Referring to figure 6, it is angle β (MQR) and will always be larger than the angle of reflection α at P from the dipping reflector.

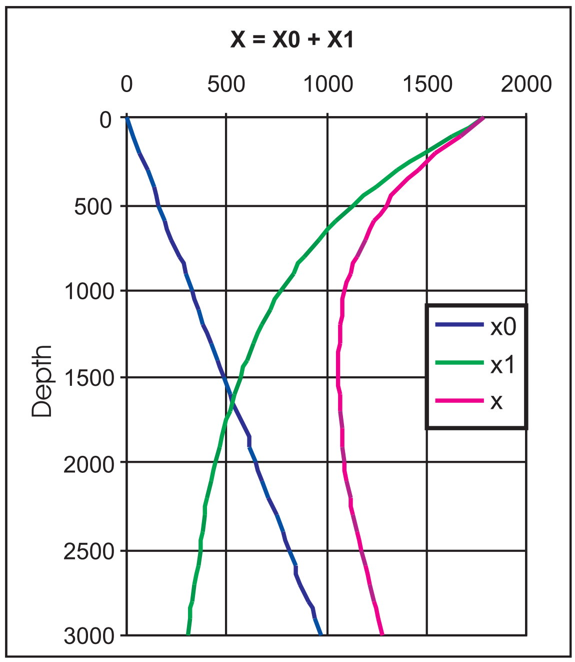

Fig.8. Depth Vs migration distance plot.

The zero offset contribution x0, offset contribution x1 and the total migration distance x are shown separately in figure 8. Notice the minimum value at Z_m = 1456.

This is due the fact that the offset contribution to migration distance (DMO part) is maximum for z=0 and is equal to half offset h and decreases with increasing depth while the zero offset contribution is zero at zero depth and increases continuously. At a specific depth the increase in zero offset contribution takes over the decreasing offset contribution.

The migrated depth vs migration distance data in the above table is pictorially represented in figure 9 below along with the curve joining the reflecting points. As all the reflection events for this source-receiver pair are recorded below their midpoint M, during migration, if one chooses a constant dip cut-off for restricting the migration distance for all depths, this curve represents the locus of the elliptical impulse response cut-off points.

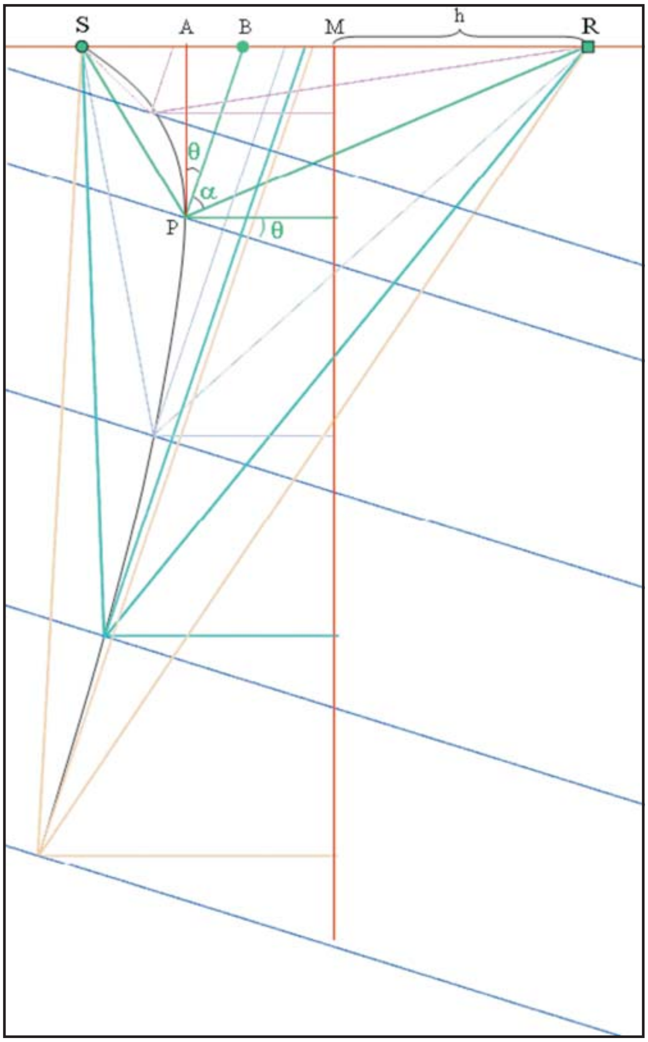

Fig.9. Locus of reflection points from parallel reflectors at different depths.

We derived the equation for the locus of the migration distance with respect to depth Z_m and found the minimum migration distance by equating its1st derivative to zero. The minimum migration distance is related to the half offset h and the dip of the reflector as given below.

$$Z_m\,(\min) = h \cos 2\theta \qquad \text{............(16)}$$

This particular depth has a physical significance. Look at the angle of reflection α at the point of reflection P from the dipping reflector in figure 9. The relation between Z_m, h, θ and α is

$$\tan(\alpha-\theta) + \tan(\alpha+\theta) = 2h/ Z_m = 2n \quad \text{............(17)}$$

Corresponding to the depth Z_m at minimum migration distance, the angle of reflection α turns out to be 45^0 irrespective of the dip of the reflector θ. For angles of incidence greater than 45^0, normally the reflection events are not recorded. This means that for all practical purposes, we get reflections from depths deeper than the depth at which migration distance is minimal and hence encounter depths for which the migration distance increases with depth.

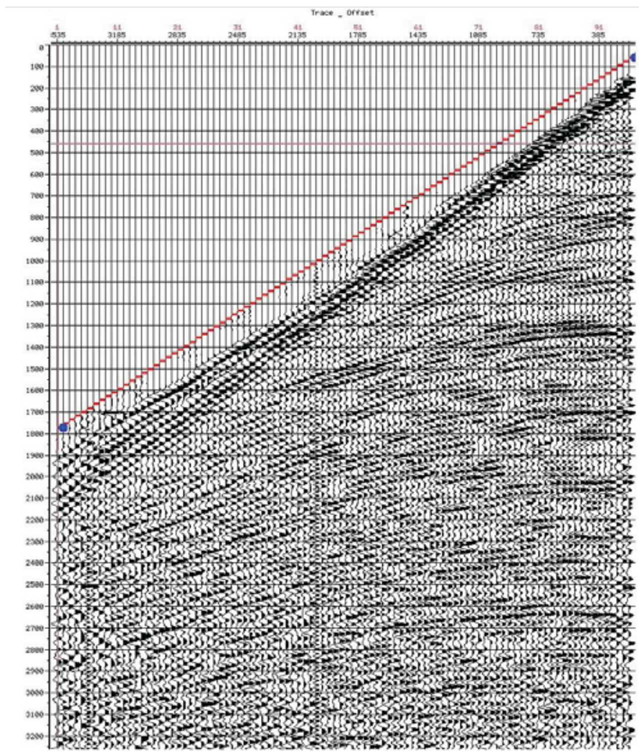

Fig.10. Shot record with far offset 3535m.

Look at the shot record where the far offset distance is 3535m and the sub-weathering velocity is close to 1900m/s. For half offset of 1767.5m we do not see any events shallower to 1900ms.

Minimum migration aperture based on Fresnel zone

The reflection energy from a point difractor is distributed along the diffraction hyperbola and needs to be summed during the migration to preserve the amplitudes. Even for a plane reflector with zero dip, the reflection energy comes from an area below the CMP position defined by the

Fresnel zone. From numerical examples, it is shown by Shung Sun et al [3] that the minimum migration aperture required to preserve the amplitudes should be twice the size of Fresnel zone area. Based on the double square root (DSR) equation, they showed that for offset section, the Fresnel zone radius is given by equation 18.

$$\sqrt{\frac{\tau v^2 \sqrt{t_0^2 + \frac{4h^2}{v^2}}}{4 - \frac{h^2}{v^2 t_0^2 + 4h^2}}} \qquad \text{...........(18)}$$

For zero offset, Fresnel zone reduces tc

the same Sheriff's definition $\frac{v}{2}\sqrt{t_0 \tau}$

Where, h – half source-receiver offset,
t_0 – two-way vertical travel time
t_h – two-way offset travel time
v – RMS velocity at the reflecting point
τ – period of the wavelet

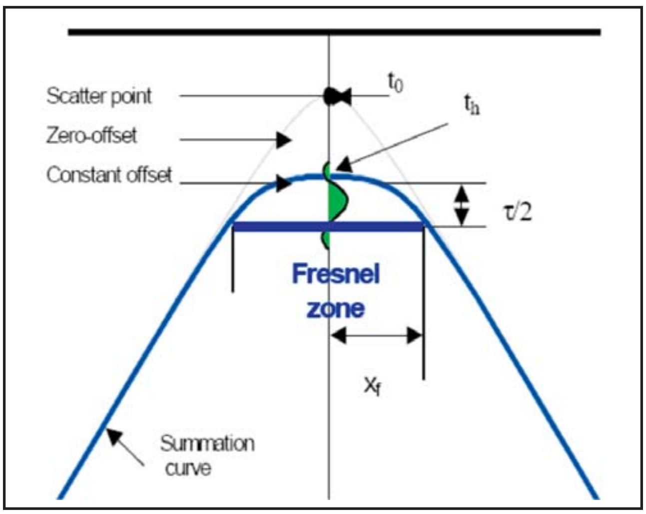

Fig.11. Fresnel Zone for offset section

Discussion

Following a different approach, earlier Srivastav et al [4] derived equation for pre-stack migration distance that can be simplified to equation 11. In this paper we established the relation between Z_m and Z_u for the non-zero offset case and obtained equations for the semi major and semi-minor axes defining the migration ellipse passing through the reflection point defined by Z_m and θ. The locus of reflection points from a series of parallel reflectors to a given source-receiver combination is the dip-cut off for the migration impulse response. It is shown that the minimum of this curve happens when the angle of incidence = angle of reflection = 45^0 irrespective of the dip of the reflector. In the next issue, we shall discuss practical consequences of migration aperture for acquisition of data.

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