Seismic Modeling in Attenuating Media
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ABSTRACT: Seismic waves attenuate and disperse as they propagate in the earth. This behavior can be modeled with an anelastic wave equation. Attenuation decreases the amplitude, narrows the bandwidth, and modifies the traveltime of the seismic waves. In anelastic materials, stress depends on the strain history, in the elastic case; stress depends on the instantaneous strain. The anelastic material behaves as if it has ‘memory’. Viscoelastic wave propagation can be modeled by incorporating the so-called memory variable in an otherwise standard finite-difference scheme. In this paper, we present implementations of a viscoacoustic and a viscoelastic 2D finite-difference scheme. We also analyze some aspects of depth imaging and amplitude versus offset (AVO) attributes using a synthetic dataset generated with our viscoacoustic algorithm. We use a model of the Valhall field that includes a gas cloud with high attenuation above the reservoir, which distorts the image and AVO attributes.

INTRODUCTION

Seismic data analysis is commonly done assuming an elastic model, however, the real earth materials disperse and attenuate seismic waves during propagation. Such media is better modeled with a viscoelastic approximation. Attenuation of propagating waves due to anelasticity can be quite significant and should ideally be compensated for in the data processing sequence (Samec and Blangy, 1992). If the an-elastic effects are not accounted for, they can be a source of errors in modeling, imaging and AVO analysis.

The seismic response is measured as amplitude and traveltime in a certain frequency band. Attenuation effects all three parameters, it 1) decreases amplitude, 2) modifies traveltime, and 3) narrows bandwidth (higher frequencies are attenuated quicker). These effects come in addition to the elastic/geometrical effects on the wavefield: geometric spreading, and scattering (reflection, refraction, and transmission). While these effects are purely elastic, the absorption is due to effects like internal frictions at grain contacts, pore fluid movement in porous rock etc. In viscoelastic media, the combined effects of all these mechanisms are modeled as a gross internal friction or intrinsic attenuation.

Modeling of wave propagation in anelastic media is performed using a linear viscoelastic model (Carcione, 2001). The viscoelastic model is a combination of viscous and elastic model, where the elastic part is responsible for the energy conservation, and the viscous part is responsible for energy attenuation. Day and Minster (1884), Robertsson et al. (1994), and Carcione (1993, 2001) have done significant work modeling in anelastic media. Here, we follow the work of Carcione, where the attenuation effects are modeled by incorporating the so-called memory variable in the finite difference scheme. These variables describe the strain history of the media. The memory variables are propagated on the model grid with the same finite-difference scheme as used for acoustic or elastic wave modeling. The attenuation model is described using a frequency independent quality factor Q, which is a measure of energy attenuation in a cycle of wave propagation; so higher Q gives lower attenuation.

In this abstract, we present the numerical implementations of a viscoacoustic and a viscoelastic finite-difference modeling algorithm. The viscoacoustic code is used to generate a synthetic data set over the Valhall model (O.Brien et al., 1999). We analyze both the acoustic and viscoacoustic data after imaging with a wavefield shot record migration. The migration algorithm does not perform for Q compensation.

METHOD

Hook’s law
\[ \sigma_{ij} = c_{ijkl} \varepsilon_{kl} \]

(1)
gives the relation between stress and strain in elastic media. In equation (1), \( \sigma_{ij} \) is the stress tensor, \( \varepsilon_{ij} \) is the strain tensor, and \( c_{ijkl} \) is the elastic tensor. Boltzmann’s superposition principle gives the relation between stress and strain in anelastic media (Carcione, 2001)

\[ \sigma_{ij} = \psi_{ijkl} * \varepsilon_{kl} \]

(2)

where \( \psi_{ijkl} \) is the relaxation tensor, and * is the convolution operation. The elastic stress-strain relationship is time independent while the anelastic constants (relaxation tensor) are time dependent. This means that in the elastic
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case, stress depends on the instantaneous strain only, hence there is no phase difference between the two. In the anelastic case, however, stress depends on the strain history, as if anelastic materials have ‘memory’, resulting in a phase difference between the stress and strain. By incorporating this memory effects in the elastic wave propagation, we get the response of anelastic wave propagation.

Acoustic wave propagation using finite-difference method is formulated as,

$$P_{t+1} = f(P_t) + 2P_t - P_{t-1}$$

where,

$$P_{t+1}$$ is the new wavefield,

$$P_t$$ the present wavefield, and

$$P_{t-1}$$ is past wavefield.

$$f(P_t) = \frac{du}{dt^2}$$, where

$$u(t)$$ is acceleration at time $$t$$, and

$$dt$$ is the sampling time interval. For the viscoacoustic case, we replace $$f(P_t)$$ with $$f(P_t+e)$$, where $$e$$ is the memory variable, which accounts for attenuation.

Following Carcione (2001), we write the memory variable in viscoacoustic media as,

$$e = [e(1-\phi)+dt.\phi pp]/(1+\phi)$$,

where,

$$\phi = \frac{1}{2}\frac{\tau_{s\upsilon}}{\tau_{s\sigma}}$$,

$$\tau_{s\sigma} = \frac{\tau_0}{Q_0}(\sqrt{1+Q_0^2}-1)$$,

$$\tau_{s\upsilon} = \frac{\tau_0}{Q_0}(\sqrt{1+Q_0^2}-1)$$,

$$\tau_0 = \frac{1}{2\pi f_0}$$.

In the above equation, $$\tau_{s\sigma}$$ and $$\tau_{s\upsilon}$$ are the stress and strain relaxation time, $$\tau_0$$ is the elastic limit of $$\tau_{s\sigma}$$ and $$\tau_{s\upsilon}$$, and $$f_0$$ is the fundamental frequency. From equation (4), we can observe that the memory will accumulate with time. In the acoustic limit ($$Q_0$$ is infinity), the memory variable $$e$$ is zero at time $$t = 0$$, hence $$\phi = 0$$ ($$\tau_{s\sigma} = \tau_{s\upsilon}$$). So the memory remains zero and the viscoacoustic formulation correctly reduces to that of acoustic media.

Elastic wave propagation in 2D uses the following stress and strain relations (Etgen, 1987),

$$\sigma_{xx} = c_{33}^0 \frac{\partial}{\partial x} u_x + c_{13}^0 \frac{\partial}{\partial z} u_z,$$

In anelastic media, the stress and strain relations can be represented by (Carcione, 1993)

$$\sigma_{xx} = c_{33}^0 \frac{\partial}{\partial x} u_x + c_{13}^0 \frac{\partial}{\partial z} u_z + [\frac{(c_{33}^0 + c_{33}^p) e_i}{\tau_{s\sigma}} + 2c_{33}^0 e_{ij}]

\sigma_{yy} = c_{33}^0 \frac{\partial}{\partial y} u_y + c_{13}^0 \frac{\partial}{\partial z} u_z + [\frac{(c_{33}^0 + c_{33}^p) e_i}{\tau_{s\upsilon}} + 2c_{33}^0 e_{ij}],

\sigma_{zz} = c_{33}^0 \frac{\partial}{\partial z} u_z + c_{13}^0 \frac{\partial}{\partial z} u_z + [\frac{(c_{33}^0 + c_{33}^p) e_i}{\tau_{s\sigma}} + 2c_{33}^0 e_{ij}].$$

(7)

In the above equations, $$\sigma_{ij}$$ is stress tensor, $$e_{ij}$$ are the memory variables, $$c_{ij}^0$$ are the elastic (relaxed) constants, $$c_{ij}^p$$ are the anelastic (unrelaxed) constants, and $$u_i$$ is the particle displacement in $$i$$th direction.

The memory variables $$e_{ij}$$ in viscoelastic media are computed using the following equations,

$$\partial \frac{t}{\tau_{s\sigma}} e = \frac{1}{\tau_{s\sigma}} [(1-\frac{\tau_0^s}{\tau_{s\sigma}})(\frac{\partial}{\partial x} u_x + \frac{\partial}{\partial y} u_y) - e_{i1}],

\partial \frac{t}{\tau_{s\upsilon}} e = \frac{1}{2\tau_{s\upsilon}} [(1-\frac{\tau_0^s}{\tau_{s\upsilon}})(\frac{\partial}{\partial y} u_y - \frac{\partial}{\partial z} u_z) - 2e_{ij}],

\partial \frac{t}{\tau_{s\sigma}} e = \frac{1}{\tau_{s\sigma}} [(1-\frac{\tau_0^p}{\tau_{s\sigma}})(\frac{\partial}{\partial z} u_z + \frac{\partial}{\partial z} u_z) - e_{i3}],$$

(8)

The anelastic constants can be written in terms of elastic (relaxed) constants as,

$$c_{33} = c_{33}^0 \frac{\tau_{s\upsilon}}{\tau_{s\sigma}},

c_{33} = (c_{33} - c_{33}^0) \frac{\tau_{s\upsilon}}{\tau_{s\sigma}} - c_{33}^0 \frac{\tau_{s\upsilon}}{\tau_{s\sigma}},

c_{33} = c_{33} - 2c_{33} = (c_{33}^0 - c_{33}) \frac{\tau_{s\upsilon}}{\tau_{s\sigma}} - c_{33}^0 \frac{\tau_{s\upsilon}}{\tau_{s\sigma}}$$

(9)

where, p and s v subscripts on $$\tau_{s\sigma}$$ and $$\tau_{s\upsilon}$$ indicate the corresponding stress and strain relaxation time for P-wave and S-wave respectively. In the viscoelastic case there are three memory variables as given in equation (8), one corresponding to P-wave attenuation, the other two for shear (S v and S H ) wave attenuation. Equation (8) is evaluated using finite difference scheme. We use a staggered grid (space and time) for the finite differences, similar to the formulations found in Robertsson et al., 1994, and Carcione, 2001.

RESULTS

The effects of attenuation are first studied in a simple homogeneous model. We consider a 6km by 6km model ($$V_p = 2000$$ m/s, and $$\rho = 2.5$$ g/cc) with an exploding source at the center of model. Figure 1 compares the
acoustic response (Figure 1a) with those of two viscoacoustic versions, Q=1000 and Q=20 respectively. High Q (Q=1000) has very little effect, as shown in Figure 1b, while strongly attenuating media (Q=20), show large effects on amplitude, phase, and frequency.

We illustrate the viscoelastic case with a similar model. The model is homogeneous in the elastic parameters (V_p = 2000 m/s, V_s = 1400 m/s, and ρ = 2.5 g/cc) but with a heterogeneity in Q. The upper half of the model has low attenuation (Q=1000), while the lower half of the model has strong attenuation (Q=20) (see Figure 2a). Figures 2(b) and 2(c) show the elastic and viscoelastic response. The effects

Figure1: Snapshots for a homogeneous acoustic model (V=2000 m/s, ρ=2.5 g/cc) at 1.2s for the point source at the center of model, a) acoustic, b) viscoacoustic (Q=1000), and c) viscoacoustic (Q=20).

Figure2: Snapshots (x component) for a homogeneous elastic model (V_p=2000 m/s, V_s=1400 m/s, ρ=2.5 g/cc) and heterogeneous Q model (a); elastic response (b), and viscoelastic response (c).
of attenuation are evident. We also observe the reflection from the Q boundary.

Finally, we analyze the effects of attenuation on depth migrated images and AVO attributes using the Valhall model (O.Brien, et. al., 1999). Figure 3(a) shows the P-wave velocity model, with low velocity gas zones above the reservoir in the high velocity chalk layer. The Q model (Figure 3b) was created using the outline of the gas cloud from the velocity model, filling the different layers with Q values in the range 1000 to 30. The lower Q values (80 to 30 from top to down) are confined to the gas zone as shown in the figure.

We have not incorporated Q compensation in the migration and AVO algorithms; instead we use two synthetic data sets, one acoustic and one viscoacoustic, to study the effects of attenuation. Both data sets are processed identically, and we compare the results difference. Figures 4(a) and 4(b) show the depth migrated images for the acoustic and viscoacoustic data respectively, and Figures 4(c) and 4(d) show reflection coefficient intercept (AVO attribute) response for the same data. We can observe higher

Figure 3: Velocity model (a) and Q model (b) for the Valhall field, used to create the shot gather for further analysis.

Figure 4: Difference in the acoustic and viscoacoustic response, a) and b) are the shot migration image with acoustic and viscoacoustic data, c) and d) are the intercept (AVO attribute) response with acoustic and viscoacoustic data.
frequency content in the acoustic data, in particular degraded resolution in the reservoir zone and below. This is illustrated in more detail in Figure 5, which shows a trace from the reservoir level for the acoustic and viscoacoustic datasets. In the viscoacoustic case (red) it is difficult to separate the thinner layers. Also, there is a 40 meters mis-tie at the reservoir level (Figure 5) due to phase distortions in the attenuating zones even when using the correct acoustic model during the imaging.

Attenuation modifies the wavefield in terms of amplitude, phase (traveltime) and frequency. Examples using the Valhall model show that the effect of attenuation impacts both resolution and imaged depth of reflectors. Therefore, in the areas with attenuation, data processing should compensate for such effects in order to obtain correct reflector depths and best possible resolution.

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REFERENCES


CONCLUSION

We show implementations of finite-difference modeling algorithms for viscoacoustic and viscoelastic media. We use the two modeling programs to study the effects of attenuation on seismic imaging and AVO analysis. The