**Optimum Time-Frequency Resolution of Seismic Data using Continuous Wavelet Transform**

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ABSTRACT: Traditionally, a time-frequency spectrum of a signal is obtained using the short-time Fourier transform (STFT). The time-frequency resolution in the STFT is fixed by the choice of a window length. However, seismic data, being non-stationary in nature, have varying spectral distribution in time. Therefore, we need a time-frequency decomposition that can provide optimum time-frequency resolution for varying frequencies. The time-frequency spectrum using the continuous wavelet transform avoids the problem of choosing a window length and provides high frequency resolution at low frequencies and high time resolution at high frequencies. We refer to such a time-frequency spectrum from the continuous wavelet transform as TFCWT. Two field examples illustrate that the TFCWT can potentially be utilized to detect frequency shadows caused by hydrocarbons and identify thin beds below the tuning thickness.

**INTRODUCTION**

Spectral decomposition of a non-stationary signal, like seismic signal, is conventionally achieved by the STFT. It produces a spectrogram whose time-frequency resolution is fixed by the choice of a window length. We avoid the problem of choosing a window length in the CWT. The CWT produces a time-scale map (also called scalogram) where the scale is defined in terms of the dilating support of a wavelet and represents a frequency band. (Daubechies, 1992; Goswami and Chan, 1999; Mallat, 1999). A scalogram does not provide a direct intuitive interpretation of frequency. In order to interpret a scalogram in terms of a time-frequency map, a number of approaches can be taken. The easiest step would be to stretch the scale to an equivalent frequency depending on the scale-frequency mapping of the wavelet. However, we take an alternative approach and analytically convert a time-scale map into a time-frequency map. Such a time-frequency map from the CWT is called TFCWT. (Sinha, 2002).

The TFCWT is particularly useful in seismic data analysis as it provides high frequency resolution at low frequencies and high time resolution at high frequencies.

**THEORY**

The time frequency spectrum using STFT is given by

\[ F_{\text{STFT}}(\omega, \tau) = \int_{-\infty}^{\infty} f(t) \bar{\phi}(t-\tau) e^{-i\omega t} dt \]  

(1)

Where, \( \bar{\phi} \) is the complex conjugate of \( \phi \), \( \tau \) is the center of the chosen time window. The CWT is defined as the inner product of a family of wavelets \( \psi_{\sigma, \tau}(t) \) with the signal \( f(t) \). This is given by

\[ F_{\psi}(\sigma, \tau) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{\sigma}} \bar{\psi}\left(\frac{t-\tau}{\sigma}\right) dt \]  

(2)

Where, \( \sigma \) is the scale, \( \bar{\psi} \) is the complex conjugate of \( \psi \), \( \tau \) is the translation parameter and \( F_{\psi}(\sigma, \tau) \) is the timescale map (i.e. scalogram). By taking the Fourier transform of the inverse continuous wavelet transform we can convert the timescale map into the time-frequency map. This is given by

\[ F_{\text{TFCWT}}(\omega, \tau) = \frac{1}{C_{\psi}} \int_{-\infty}^{\infty} F_{\psi}(\sigma, \tau)e^{i\omega\tau} \psi(\sigma \omega) \frac{d\sigma}{\sigma^2} \]  

(3)

Where, \( ^{^\wedge} \) represents a function in the Fourier domain and \( C_{\psi} \) is a wavelet dependent constant. Time-frequency map, thus produced in eq (3) is called TFCWT. Equation (3) is the fundamental result used in this work to compute time frequency spectra of seismic data.

**SYNTHETIC EXAMPLE**

In this section we compare the two methods, viz. STFT and TFCWT, using a synthetic chirp signal. The chirp signal has two hyperbolic sweep frequencies increasing with
Time-Frequency Resolution

The time-frequency map with the STFT using a 400 ms hamming window is shown in Figure 1. It indicates that the low frequency components are well resolved compared to the high frequencies since the chosen time window is too broad. Choosing a shorter window length will compromise the frequency resolution to obtain higher time resolution. Therefore, analyzing a non-stationary signal with the STFT has a practical limitation of choosing an appropriate window length. However, the time-frequency map from the TFCWT as shown in Figure 2 does not require any window length. We also observe that the time-frequency resolution is significantly better as compared to that in the Figure 1.

Figure 1: The time-frequency spectrum of a chirp signal with two hyperbolic sweep frequencies produced from the STFT using a 400 ms Hamming window.

Figure 2: The time-frequency map of a chirp signal analyzed with the TFCWT using the Morlet wavelet.

APPLICATIONS OF TFCWT TO SEISMIC DATA

Interpretation and visualization of seismic data in the frequency domain is an important tool to study the geological information in a region. Typically such analysis is carried out with post-stack data sets. In this procedure a seismic signal is mapped into time-frequency plane using either STFT or TFCWT or other spectral decomposition method. In the time-frequency space, interpretation can be made by taking a single frequency amplitude or power as the seismic attribute. Two different ways of interpreting seismic data in frequency space are illustrated.

A. Single Frequency Seismic (SFS) section

Time-frequency analysis of a one dimensional trace produces two dimensional data set by adding a frequency axis and a two dimensional seismic section generates a 3D data volume with the third axis being frequency up to the Nyquist. Any section at a single frequency from this 3D data volume is called a single frequency seismic (SFS) section. Comparison of different SFS sections can be utilized to detect low frequency shadows caused by hydrocarbon reservoirs. This method can potentially be utilized for direct hydrocarbon detection (Sun, Castagna and Siegfried, 2002).

A seismic section from Ukpokiti, Nigeria shown in Figure 3 is interpreted using this method. Bright amplitudes in the data are indicative of hydrocarbon zones. An SFS section at 20 Hz from the TFCWT data volume shows high amplitude low frequency anomalies (colored as red) at the reservoir level in Figure 4. At 33 Hz these anomalies disappear as shown in Figure 5. The anomaly above the hydrocarbon reservoir level in 33 Hz section is due to local tuning effect which does not disappear at higher frequencies. This example shows that the comparison of SFS sections from TFCWT have been able to detect low frequency shadows caused by hydrocarbon.

B. Single Time-Frequency (STF) Slice

Addition of frequency axis to a 3D seismic data volume makes the time-frequency volume 4D and makes the visualization difficult. To make visualization simple, a 3D seismic data volume can be rearranged in 2D according to trace number or CDP. Time-frequency analysis on this data will extend it in the third dimension adding a frequency axis
Figure 3: A seismic section from Ukpokiti, Nigeria showing hydrocarbon zones as bright amplitudes adjacent to faults.

to it. From such a time-frequency-CDP volume, we can take a slice at fixed time and rearrange the trace numbers according to their inline and crossline numbers to produce a frequency-space cube. Visualization of frequency slices (i.e. single time-frequency slices) from such a 3D cube can be utilized for thin beds identification and reservoir characterization.

A time-slice has been taken from the Waha-Lockridge 3D seismic data volume shown in Figure 6. A channel feature is indicated in blue. An important geological question is: whether the channel ends in the middle of the area or is it beyond the resolution capability of the seismic because of thin bed? From an interpreter’s point of view the extension of this channel feature is crucial information for reservoir characterization. A 20 Hz frequency slice for the same time

Figure 4: A 20 Hz single frequency seismic section showing anomalies at the hydrocarbon reservoir levels.

Figure 5: Low frequency high amplitude anomaly in Figure 4 disappeared in this 33 Hz SFS section. Anomaly in this figure is due to tuning of thin beds.

Figure 6: A time slice from Waha-Lockridge 3D data volume showing a channel feature in blue

Figure 7: This is a 20 Hz STF slice. The channel feature is not thick enough to tune at 20 Hz.
slice presented in Figure 7 is not indicative of the channel. A possible interpretation would be that the channel is not thick enough to tune at 20 Hz. For the same time-slice the 40 Hz section in Figure 8 shows the channel feature similar to the amplitude section in Figure 6. It is important to note that the dominant frequency of the seismic data is about 40 Hz. Therefore, the 40 Hz slice shows similar feature as the amplitude section. At 95 Hz time slice shown in Figure 9, the channel feature is greatly enhanced and we can see a thin meandering channel at the bottom center. Such analysis is important for reservoir characterization to constrain fluid flow for reservoir simulation studies.

CONCLUSION

In this paper we have presented a new method (TFCWT) to compute time frequency spectrum using wavelets as adaptive window. Conventional method like STFT has inherent drawback of selecting a window length that makes the processing and interpretation subjective. The TFCWT method overcomes this problem and gives a more robust technique for time-frequency attribute analysis. The dilation and compression of the wavelets effectively provides the optimal window length depending upon the frequency content of the signal. Hence, the TFCWT provides optimal time-frequency resolution. The computation of the CWT in the Fourier domain provides the flexibility to compute the time frequency map without much effort. Synthetic example presented in the paper shows the advantage of our method over the STFT. The field examples on frequency attribute computed using TFCWT presented in this work provide optimism that single frequency sections can potentially be utilized as a direct hydrocarbon indicator and single time-frequency slices can be used to enhance thin bed reservoirs.

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