The pressure-driven flow simulations on 3D micro-CT data using Lattice-Boltzmann method

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Keywords
Permeability, Lattice-Boltzmann, microCT, Darcy’s law, pressure-driven flow

Summary
Carbonate reservoirs exhibit heterogeneity due to the variation in porosity types and their complex network. This causes a change in the permeability of the rock. Since most of the hydrocarbon reserves are located in carbonate reservoirs, the study of permeability is extremely important. Here we simulate the pressure-driven flow on high-resolution microCT images of core samples using the Lattice-Boltzmann (LBM) method. The viscosity dependent permeability is simulated using the D3Q19 model with multiple-relaxation time. The permeability was calculated using the Darcy’s law in the steady-state conditions. The simulated results agree with those obtained from the experiments for the same core sample.

Introduction
Several attempts have been made to develop pore-scale models to study the micro-scale behavior in carbonates. These models are used to represent the micro-scale system, from which, macro-scale behavior can help in upscaling (Blunt 2001; Nunes et al. 2016; Khishvand et al 2016; Christie et al 2001) from core scale obtained from coring of wells to the reservoir level. Carbonates exhibit porosities at various scales. This goes down to the nanoscale range like the shales where most of the porosity is due to the nano-sized pores. Kazemi et al. (2016) used the nanopore data for upscaling and modelling their shale data. Despite so many studies, user subjectivity and control are required in these modelling approaches. Thus, in the current work we perform fluid-flow modelling using the LBM unlike the conventionally used Finite Element Model (FEM) approaches.

LBM is a recent development and is being adopted for performing fluid. Here we model the Darcy flow problem for an arbitrary geometry exhibited by the carbonate rock core sample. This is primarily achieved using Digital Image Processing to treat raw 3D-micro Computed Tomography (3D-µCT) images of core samples from the carbonate reservoirs followed by the application of LBM method for Darcy equation on the treated 3D-µCT image data.

Methodology
Before performing the actual simulation on real core sample, a qualitative study was done to check the working of the code. This 3D path was formed from a 2D image by stacking the image 40 times along the X-axis. The sample 2D image is shown in Fig. 1 while the corresponding 3D path is shown in Fig. 2.

Figure 1. An arbitrary 2D flow path.
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Figure 2. The 3D path formed by stacking the 2D images of the arbitrary path

Permeability was simulated using the LBM methodology on this arbitrary 3D path sample. The results of the simulation are shown in Fig. 3 from two different viewing angles. The permeability calculated on the 3D sample was 0.28 lattice permeability units with an average velocity of 4.29e-4 lattice velocity units.

Data Preparation

Next, the study was performed on a sample of carbonate core sample obtained from the reservoir. Data preparation constituted several preprocessing stages like data acquisition, data loading, histogram equalization and 3D image binarization.

a) Data acquisition
The carbonate rock samples were obtained in the form of cores. The samples had been numbered depending on the well they were extracted from. These raw rock samples were subjected to Four-Dimensional X-Ray Micro tomography (FDXM) facility in the institute for internal imaging in the micron level scale (1 pixel = 10 µm). The resolution of the equipment was set to 10µm for the imaging purpose. This means that each image captured had a square pixel dimension of 10µm. Samples were cleaned and carved in the form of cube of 0.7cm length. On testing these samples in the FDXM facility the voxelised image obtained was of 1024 × 1004 × 1016 dimension each. This 3D image was further cropped to 324 × 300 × 314 voxelised image to quicken the analysis, as working with 3D data is too resource intensive. The 3D cubes obtained were further processed using image processing techniques.

b) Data loading
A Python program was written to read Object Linked Embedding (OLE) object which is the standard container by Microsoft. The program helped read ‘.txm’ files which were natively output by the µCT equipment.

c) Histogram equalization
The 3D cube loaded after the imaging generally contain irregular intensity, i.e. to say, the pixels
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making the intensity map of the image do not cover the entire intensity range (say 0-255 for unsigned integer datatype) making it difficult to identify the porous part from the non-porous parts (Fig. 4).

![Image](image_url)

**Figure 4.** Histogram Equalization of a core sample.

A comparison of the same is shown in Fig. 2 where we can easily see the features have been rediscovered by the algorithm.

d) 3D Image Binarization

It is required to set a threshold for separating the matrix from the porous part. A suitable threshold level is chosen for each of the 3D images such that the simulated porosity matches with the experimental porosity.

This way the simulated model can be calibrated against the experimental porosity. This global parameter is highly deterministic about the porosity of the sample under test so the previous step of histogram equalization (before binarization) must be diligently handled.

Binarization is the process of selection of all the pixels holding a value less than the threshold are assigned as 0 and the pixels other than this as 1. This way the image now holds only two intensity values, hence the name binary. The above-mentioned steps can be figuratively summarized below (Fig. 5).

![Image](image_url)

**Figure 5.** The Image processing of the µCT of the core sample.

Lattice Boltzmann Simulation in porous media

The Lattice Boltzmann methods (Shan 1993) for fluid simulation have seen a lot of development in the past decade. They are not only easy to parallelize but also offer flexibility in modelling various types of flows. They include porous media flows, multiphase flows, magnetic and particulate suspensions. In all these problems the one has to deal with complex boundary conditions and variable Reynolds number. When compared with the conventional PDE algorithms, such as FVMs, LBMs have the strength to leverage the computing power of graphics processing units (GPUs).

In this work the LBM method has been used to model the Darcy flow to evaluate the permeability of 4 carbonate rock samples from the Mumbai High region. A D3Q19 model is used for the fluid emulating particles at each individual lattice locations, x at time t; In the density function \( f_a(x, t) \), \( a \) denotes the 19 directions for the fluid to move towards and \( e_a \) the corresponding velocities:

\[
[(0,0,0), (1,0,0), (-1,0,0), (0,1,0), (0,-1,0), (0,0,1),
(0,0,-1), (1,1,0), (1,-1,0), (-1,1,0), (-1,-1,0), (1,0,1),
(-1,0,1), (1,0,-1), (-1,0,-1), (0,1,1), (0,1,-1), (0,-1,1),
(0,-1,-1)]
\]

The process proceeds by performing at each time step \( t \) a collision between the fluid particles which redistributes the density distribution to a local equilibrium. Next in the step comes the streaming wherein at the neighboring location designated by \( x + e_a \Delta t \), the distribution functions get shifted. Here \( e_a \) are the direction velocities; The process can be represented by:
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\[
f_a(x + \vec{v}_a, t + \Delta t) = f_a(x, t) + \tau^{-1} \left( f_a^{eq}(x, t) - f_a(x, t) \right) + \frac{\Delta t}{F_a}
\]

Here \( \tau \) denotes the fluid relaxation time and \( f_a^{eq}(x, t) \) represents the Maxwell Boltzmann distribution function given by:

\[
f_a^{eq}(x, t) = w_a \rho(x) \left[ 1 + \frac{\vec{v}_a \cdot \vec{u}}{c_a^2} + \frac{9}{2} \frac{(\vec{v}_a \cdot \vec{u})^2}{c_a^4} - \frac{1}{2} \frac{(\vec{u})^2}{c^2} \right],
\]

with the weights \( w_a = \frac{1}{3} \) for \( a = 0 \), \( w_a = \frac{1}{18} \) for \( a = 1, \ldots, 6 \) and \( w_a = \frac{1}{36} \) for \( a = 7, \ldots, 18 \); \( c_a \) here represents the sound speed.

The following equation gives the relation with the force applied:

\[
F_a(x, t) = w_a (1 - \frac{1}{2\tau}) (3 \frac{\vec{v}_a - \vec{u}}{c^2} + 9 \frac{\vec{v}_a (\vec{v}_a \cdot \vec{u})}{c^4}) \vec{F}
\]

The fluid properties of interest like velocity (\( \vec{u} \)) and density (\( \rho \)) can be determined using equations:

\[
\rho = \sum_{a=0}^{18} f_a \quad \text{and} \quad \vec{u} = \frac{1}{\rho} \sum_{a=0}^{18} f_a \vec{v}_a + \frac{\vec{F}_a}{2\rho}.
\]

LBM Formulation of Darcy Flow: Lattice Boltzmann (LB) method was used to compute permeability (\( k \)) of each axis. A \((DQ19)\) model with multiple relaxation time was implemented in the opensource LBM package Palabos (Chopard, 2009). This is the preferred approach to single-relaxation time when using for complex geometries (Hilpert, 2011). For the inlet and outlet, a fixed pressure boundary condition was imposed. No slip boundary condition was imposed on the four sides perpendicular to the main flow direction and a simple bounce-back boundary condition was imposed on solid boundaries obtained from the segmented imaging results. Darcy law falls within the laminar flow regime and all simulations were carried out to honor that. Each directional permeability was evaluated by using the total flux to the pressure gradient using the equation:

\[
U = -\frac{k}{\mu} \frac{dP}{dx}
\]

where \( U \) is the average velocity, \( \mu \) is the fluid viscosity, \(-dP/dx\) is the pressure gradient \((=dP/L)\) where \( L \) is the length in the main flow direction), with \( x \) being the flow direction.

Permeability Simulation on core

Mentioned are the boundary conditions and modelling strategies used for the cores. Pressure difference between inlet and outlet was kept as \( 1.5 \times 10^{-5} \) \( \text{atm} \). A bounce-back boundary condition was applied at the walls.

**Lattice units: Physical units = 1 \( \text{lu} : 10 \text{um} \)**

The whole domain was binarized such that the voxel held a True value where the fluid was supposed to flow and a False value at the matrix region. For each of the three axes the inlet and outlet were designated along the flow direction such that they were perpendicular to the path. The simulations were then run until convergence. Number of iterations were fixed at 10000.
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Figure 6. The Darcy Flow along x-axis (a), y-axis (b) and z-axis (c)

Results

A comparison table of experimentally derived permeability and numerically derived permeability is shown in Table 1. Reported are the directional permeabilities along the x, y and z axes. The calibration table for porosity is shown in Table 2.

<table>
<thead>
<tr>
<th>Permeability</th>
<th>Sample#105</th>
</tr>
</thead>
<tbody>
<tr>
<td>K experimental</td>
<td>( k = 58.10 ) mD</td>
</tr>
<tr>
<td>K simulation</td>
<td>( k_x = 63.30 ) mD</td>
</tr>
<tr>
<td></td>
<td>( k_y = 49.59 ) mD</td>
</tr>
<tr>
<td></td>
<td>( k_z = 11.58 ) mD</td>
</tr>
</tbody>
</table>

Table 1: Simulated and experimental permeability value

Table 2: Porosity determined from experiment and those calibrated from experiment

<table>
<thead>
<tr>
<th>Porosity</th>
<th>Sample#105</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi ) experimental</td>
<td>24.50 %</td>
</tr>
<tr>
<td>( \Phi ) after calibration from experimental</td>
<td>23.49 %</td>
</tr>
</tbody>
</table>

Conclusions

The LBM was used to perform fluid flow on 3D \( \mu \)CT data obtained from carbonate sample of the Mumbai offshore field. The stages of processing comprise of digital image processing and fluid flow modelling. The following can be concluded from the study:

1) Image processing is an essential step for quality check of the data. It is difficult to set a global thresholding parameter, which honor the porosity and permeability at the same time.
2) The LBM method is a simple yet sophisticated fluid flow solver as far as its application is concerned. The boundary conditions can be easily applied by merely changing the reference-id of the boundary lattice points, such as integer 1 for boundaries and integer 0 for the domain where the fluid needs to flow.
3) The porosity and permeability predicted from LBM simulation agree well with those obtained from laboratory measurement of these parameters.

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Acknowledgments

The authors acknowledge GEOPIC, ONGC for kindly providing the log data and core samples through PAN-IIT research program.