Identifying Fracture Orientations with Volume Based Curvature

Mumbai High

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Summary

We use curvature data to illustrate subtle structures throughout the 3-D seismic data over the Mumbai High and relate these structures to the locations of fractures. Curvature illuminates relative changes in the dip of formations and indicate that such changes in dip may cause brittle rocks, such as carbonates and igneous basement rocks, to fracture. In this area, lineaments in the maximum curvature data at the L3 carbonate level tend to be relatively short in length at a “long wavelength” resolution (100 m), but much longer at the “SubRegional” resolution (400 m). At the longer wavelength, the Permeability Barrier separating the northern part of the Mumbai High field from the southern part is much better defined. At the basement level, the shorter wavelength data shows better lateral continuity, suggesting that the structures at this level are narrower.

Introduction

Curvature is a mathematical technique for analyzing the nature of surfaces. We apply this technique to geologic surfaces in the form of 3-D seismic data to analyze formation interfaces. We postulate that a change in the dip of a geologic surface is likely to cause fracturing of brittle rock. Curvature manifests itself on time-, depth-, and horizon-slice as lineaments representing subtle anticlines, synclines, faults, and flexures, often showing (locally) preferred orientations, suggesting that the fractures in these areas may follow those same orientations.

Curvature

Basic Theory

Roberts (1971) reports that Gauss first studied curvature of surfaces in the 1820’s. In mathematical terms, curvature is related to the second derivative of the mathematical equation describing the surface. In simpler terms, curvature is the inverse of the radius of a tangent circle. Figure 1 illustrates curvature in a 2-D sense. Along a smoothly varying line, there will be areas where the line is concave downward. In geologic terms, this represents antclinal folding. In other areas, the line is concave upward, corresponding to synclinal folding. These correspond to positive and negative values of curvature, respectively. As the folding gets broader, the tangent circle and its radius get larger until line is straight and the radius is infinite. In terms of curvature, this corresponds to decreasing values of curvature with the limiting case being a magnitude of 0.

In three dimensions, the situation is more complicated (Figure 2). Curvature is a two dimensional attribute and, for any Point P, the orientation of the tangent circle may be along any azimuth (in geologic sense) between $0^\circ$ and $180^\circ$. From Gaussian theory, there are two Principal Curvatures, the maximum curvature ($k_{\text{max}}$) which corresponds to the smallest tangent circle and the minimum curvature ($k_{\text{min}}$) which corresponds to the broadest tangent circle. In the case of quadratic or second order surfaces as Gauss used, these two Principal Curvatures are perpendicular to one another. The product of the maximum curvature and the minimum curvature is the Gaussian curvature ($k_G$).
Addition useful curvatures are those oriented along what in
géologic terms are the dip (\(k_{\text{sp}}\)) and strike (\(k_{\text{str}}\))
directions, and the most positive (\(k_{\text{pos}}\)) and most negative
(\(k_{\text{neg}}\)) curvatures which are corollaries of the maximum
curvature.

The Gaussian curvature has special properties which
previous workers (e.g., Murray, 1968, and Hennings et al.,
2003) have used to determine locations of increased
fracture density along geologic surfaces. Figure 3B shows
an initially flat surface being folded in a cylindrical or
conical manner. In such a case, there is always an axis
along which there is no folding and the curvature along this
axis is 0. This is the minimum curvature and the
respective Gaussian curvature, for which minimum
curvature is a factor, is 0. Conversely, for structures such
as domes, bowls, and saddles (Figure 3B), there is no axis
along which curvature is 0. Hence, Gaussian curvature is
non-zero. Such structures cannot be formed from
cylindrical folding of an initially flat surface. Such a
surface must be stretched, broken, or otherwise deformed to
form one of these structures. If the surface being folded is
a brittle rock layer, such as a carbonate, it is likely to be
fractured in the areas of those structures. Therefore, areas
of non-zero Gaussian curvature are likely to be areas of
increased fracturing.

The Gaussian curvature, however, may not identify all the
areas of fracturing. Figure 4 shows that a thin slab subject
to isometric folding may also have fractures. The red line
in the figure illustrates the crest of the structure which is
generally the area of tightest folding. Within such a slab,
there is a neutral surface above which the slab is in
extension and below which the slab is in compression
(Roberts, 2001). In the case of rock volumes, layers are
generally very thin relative to the radius of curvature and
they tend to slip along bedding planes such that the entire
section is in extension. When the rock volume is brittle, it
fractures under these conditions, with the fractures parallel
to the axis of the structure. Since the minimum curvature,
which is oriented parallel to the structural axis, is 0 in this
case, Gaussian curvature does not identify these areas.
However, maximum curvature does illuminate them.

Maximum curvature has another use for interpretation
(Figure 5). A flexure may be thought of as a slab which
initially folds down and then flattens out. Under this
definition, the initial fold produces an axis of positive
curvature values and the flattening out produces an axis of
negative curvature values. Therefore, in map view, such a
flexure appears as a pair of parallel lines, one of positive
values and one of negative values, with the axis of positive
values indicating the upthrown side. This relation also
exists in the case in which the flexure is broken to form a
fault. In geologic situations, the identification of faulting is
often a critical factor. The use of seismic data in general
and coherency data in particular has been a valuable aid to
this process. However, seismic data is extremely coarse in
its vertical resolution and the question of the actual lateral
extent of a fault is difficult to answer. Maximum curvature

provides a means to extend fault traces beyond the limits
normally imposed by seismic data.

Maximum curvature has two attributes which limit its use.
In the first place, in areas in which an anticline and a
syncline intersect, maximum curvature only illuminates one
of the trends, the one whose magnitude is greater,
consequently, one trend may seem to truncate the other.
This problem is resolved by the most positive and most
negative curvatures (Figure 6). These attributes result from
calculations which identify the largest positive magnitude
and the largest negative magnitude, respectively, at each
point. They are not simply a variation of the maximum
curvature in which one polarity or the other is “zeroed out”.
The overall effect is that most positive curvature
emphasizes anticlines and the most negative emphasizes
synclines. This results in lineaments being more obvious
on map view displays of most positive or most negative
curvature than on maximum curvature.

The second issue with maximum curvature that is resolved
by most positive or most negative curvature is structural
ambiguity. Structures fall into three general categories:
anticlines, which have a single axis, synclines, which have
a single axis, and flexures, which have two axes. Map
views of maximum curvature do not distinguish between
these three categories. The other two attributes, by virtue
of showing only one polarity of curvature reduce all
structures to a single axis.

The final issue about curvature to be discussed here is that
curvatures is function of lateral resolution or scale. At any
point in the earth, individual grains have curvature (Figure
7) and the earth itself has curvature. Additionally, various
tectonic processes create structures of different size that lie
between the end members described above. Each of these
types of structures has curvature which may be resolved
with a different aperture during the curvature computations.

**Surface versus Volume Curvature**

As noted earlier, curvature requires a surface that is
analyzed. The problem for geologists has always been how
to define the surface. Contouring or curve fitting discrete
points predetermines the curvature results. Lisle (1994)
solved this problem by recognizing that 3-D seismic data
provides a naturally gridded surface. He developed a
method of determining Gaussian curvature (Figure 8) from
such a gridded surface. This method is somewhat labor
intensive and provides only the Gaussian curvature. As we
have seen above, other curvatures may yield useful
information about fractures.

Roberts (2001) extended this concept. He observed that,
for any given point, the coefficients of the best fit quadratic
surface (\(z = ax^2 + by^2 + cxy + dx + ey\)) may be determined
from the values at that point and the eight adjacent points.
He then showed how, from these coefficients, he could
calculate all the various curvatures. Therefore, the
computation could be automated and all inclusive.
With seismic data, however, there are numerous problems with autopicking. Figure 9 illustrates the problem when a horizon is picked along a peak. The erratic nature of the picks affects the curvature computations by introducing high frequency variations that are not associated with the real geology of the structure. The volume based method uses a small subvolume, generally about one wavelength of the data vertically and three traces by three traces laterally, with which to determine the strike and dip of the formations. This provides much smoother surfaces for input to the curvature calculations and therefore superior curvature results. The volumetric method also permits display of curvature along time slices and along horizon slices parallel to some marker horizon in areas in which continuous horizons are not obvious. The method also permits display of curvature values in vertical section to help in understanding of continuity of fractures between horizons.

**Interpretation**

The data set which we processed through volume based curvature consists of a 12.5 km by 12.5 km section of a survey covering the Mumbai high. Figure 10 shows line 5400 from this survey with the L2, L3, and Basement horizons annotated on it. Figure 11 shows the Time Slice at 1440 ms from the conventional data set. The Permeability barrier separating the northern part of the field from the southern part of the field is annotated on this section. Coherency data (Figure 12) at the same level shows better definition of the edges of the Permeability Barrier and suggests a second, similar feature oriented in a more northeasterly-southwesterly direction.

Figure 13 shows the maximum curvature at long wavelength (100 m) resolution along the same time slice as Figures 11 and 12. The edges of the Permeability Barrier are somewhat well defined. Additionally, numerous lineaments defining small blocks are visible and these lineaments appear to have a limited number of orientations. An enlargement of the northeast corner of this figure (Figure 14) show that these lineaments generally consist of pairs of parallel blue (positive) and brown (negative) lines. Figure 15 shows the most positive curvature for the 1.440 second time slice. In this case, the boundaries of the Permeability Barrier are very well defined, and the secondary trend is also visible. The overall density of lineaments is less than for the maximum curvature, but the continuity is increased. The density of the lineaments within the Permeability Barrier is much less than the density of lineaments outside the zone, and this density may be related to the nature of that zone. An enlargement of the northeast corner (Figure 16) shows the increase in continuity of the lineaments, and shows some variation in density of those lineaments in the area outside of the Permeability Barrier. Figure 17 shows the Sub-Regional wavelength (400 m) of the maximum curvature. The edges of the Permeability Barrier are much better defined than for the long wavelength version (Figure 13), the overall density of lineaments is much less, and the continuity is better. This suggests that the Sub-Regional wavelength data images larger structures.

Figures 18 and 19 show the maximum curvature at long and SubRegional wavelengths for a time slice near the basement horizon (1700 ms). Comparison of Figures 13 and 18 shows that the locations and patterns of lineaments at the two different levels are different. At the deeper level, there is a well defined series of lineaments trending north-northeast to south-southwest. At this level, however, the broader SubRegional wavelength data shows less continuity than the more limited long wavelength data, indicating that the structures generating the curvature anomalies at this level are narrower, but more continuous than those at the shallower level.

**Conclusions**

We have shown that subtle structures exist in this seismic data set that are not resolvable by normal seismic data, but are defined on curvature data as lineaments. These structures have variable orientations at different levels of the data.

**References**

Hennings, P. H., J. E. Olson, and L. B. Thompson, 2003, Combining outcrop data and three-dimensional structural models to characterize fractured reservoirs: An example from Wyoming, Association of Petroleum Geologists Bulletin, V. 84, No. 6, p. 830-849


Lisle, R. J., 1992, Constant bed-length folding: three-dimensional geometrical implications, Journal of Structural Geology, V. 14, p. 245-252

Figure 1. Curvature along a line. Curvature, $k$, is the inverse of the radius, $R$, of the tangent circle at every point along the line. Where the line is concave downward (anticlinal folding) the curvature values are positive and where the line is concave upward (synclinal folding) the curvature values are negative. As the folding becomes sharper, the tangent circles become smaller, and the curvatures become larger.
Figure 2. Curvature in three dimensions. In the three-dimensional case, there are an essentially infinite number of orientations, from $0^\circ$ to $180^\circ$, along which to draw the tangent circle. The two Principal Curvatures are for the “tightest” folding (maximum curvature, $k_{\text{max}}$) and the broadest folding (minimum curvature, $k_{\text{min}}$), which are orthogonal. Additional curvatures are the curvature in the dip direction ($k_d$), the curvature in the strike direction ($k_s$), and the most positive and most negative curvatures which are corollaries of the maximum curvature.

Gaussian Curvature

$$k_g = k_{\text{max}} \times k_{\text{min}}$$

after Wynn and Stewart, 2003
Figure 3. Uses of Gaussian curvature. Folding of an initially flat slab in a cylindrical or conical manner (A) always results in an axis which is unfolded and along which the curvature is 0. This corresponds to the minimum curvature and the Gaussian curvature in all such cases is 0. Such folding is called isometric. Creation of more complex structures such as domes, bowl, saddles, etc. (B), cannot result from isometric folding. An initially flat surface must be stretched, broken, torn, or otherwise deformed to produce such a result, and the resulting surface has non-zero minimum curvature. The Gaussian curvature is therefore non-zero. Thus, areas on a geologic surface with non-zero Gaussian curvature are the most likely locations for fractures.

Figure 4. Flexing of a thin slab. When a thin slab is bent during stress, there is a structural axis (red line) which corresponds to the points of greatest folding. The upper part of this slab is in extension, which results in fractures parallel to the fold axis. The lower part of the slab is in compression. However, in nature, such slabs exhibit bedding plane slip such that the thickness of the slab approaches zero and all parts are in extension. Perpendicular to the structural axis curvature values are a maximum and parallel to this axis curvature values are a minimum.
Figure 5. Maximum Curvature. In map view, a flexure appears as a pair of lineaments, one of positive values and one on negative values, representing the two hinges of the flexure. The “upthrown” side of the flexure corresponds to the lineament of positive values. This same relation exists for faults.

Figure 6. Most Positive and Most Negative Curvatures. In some cases such as saddles, there are both positive and negative values of curvature at a particular point. Maximum curvature returns the value with the largest magnitude. Therefore, in a saddle, which may be thought of as the intersection of an anticline and a syncline, one of those structures will be illuminated at the expense of the other. Most positive and most negative curvatures solve that problem by emphasizing anticlines and synclines, respectively.

\[ k_1 = k_{\text{min}} < 0 = k_{\text{neg}} < 0 \]
\[ k_2 = k_{\text{max}} > 0 = k_{\text{pos}} > 0 \]
Figures 7. Scale dependence of curvature. At every point in the earth, grains have curvature and the earth itself has curvature. In between these two extremes, many other stresses create structures, each of which has its own curvature and corresponding fractures.
Figure 8. Curvature from a gridded surface. Analysis of the elevation of a point as compared to the four points around it determines the Gaussian curvature. The method is somewhat labor intensive and yields only the Gaussian curvature.

Figure 9. Autopicking of seismic data. The autopicked horizons in red and magenta illustrate variations which are not likely to exist in the geology of the structure. The small box illustrates the size of a subvolume used to determine the dip at the center of the box. Use of this subvolume effectively results in horizon picking using a wavelength of the seismic data, rather than focusing on a single peak or trough.
Figure 10. Line 5400 from Mumbai High 3-D survey. Section is in two way time. Approximate vertical exaggeration is 5x.

Figure 11. Conventional seismic data at Time 1.440 sec. Slice is within the L3 carbonate Permeability Barrier.
Figure 12. Coherency data at Time 1.440 sec. Slice is within the L3 carbonate. The edges of the Permeability Barrier are more obvious in this display.

Figure 13. Maximum curvature at Time 1.440 sec. Slice is within the L3 carbonate. The edges of the Permeability Barrier may be seen as relatively well defined segments. Also visible on this display are numerous lineament bounded sub-blocks.
Figure 14. Enlargement of northeast corner of figure 13. Positive values shown in blue, negative values in brown. Numerous lineaments indicating faults or flexures may be seen.

Figure 15. Most positive curvature at Time 1.440 sec. Slice is within the L3 carbonate. The edges of the Permeability are very well define and the possible secondary feature appears better resolved. The subsidiary lineaments appear as individual lineaments rather than small block boundaries and show less density in the Permeability Barrier. Compare to Figure 13.
Figure 16. Enlargement of northeast corner of figure 15. Positive values are shown in black, and negative values are not shown, reducing the ambiguity over the number of structures.

Figure 17. Maximum curvature (SubRegional Resolution) at Time 1.440 sec. Permeability Barrier is much better defined and number of lineaments is reduced in comparison to the maximum curvature at long wavelength resolution (Figure 13).
Figure 18. Maximum curvature at Time 1.700 sec. Slice is at the approximate basement. Permeability Barrier is well defined and a north-northeast to south-southwest trend appears.

Figure 19. Maximum curvature at Time 1.700 sec., Sub Regional resolution. Density of lineaments is reduced, but continuity of some is increased. Other lineaments which are visible at the long wavelength resolution (Figure 17) are not visible here, suggesting their lateral resolution is relatively small.