Summary

The recent work of Goloshubin et. al. and Silin et. al. [1][2][3] shows that the reflection coefficient is a function of frequency in fluid saturated media. In fact, they have derived a formula for the reflection coefficient as a function of frequency in the low frequency regime. For typical values of parameters, the reflection coefficient is higher at the low frequency end of this regime. We have studied the reflection coefficient as a function of frequency on real data with well control and found good correlation with well data. We have modified Silin’s formula by including the effect of tuning, and plotted reflection coefficient as a function of frequency.

Introduction

It was known for some time that reflection from the boundary between an elastic medium and a fluid saturated medium is frequency dependent[4][5]. However, Silin et. al. have been the first to make a detailed study of this phenomenon and obtain formulae for reflection coefficient and attenuation factor in the low frequency regime. They have further shown that a study of the low frequency behaviour of the reflection coefficient serves as a reliable tool for predicting fluid/hydrocarbon presence.

Encouraged by their results, we have carried out a preliminary study on real data with well control to see if we could get similar results. Our study seems to corroborate their findings. We have also modified Silin et. al’s asymptotic formula for reflection coefficient by incorporating the effect of tuning. We have obtained the ratio of the instantaneous amplitude at the reflector and the input wavelet amplitude as a function of frequency. The qualitative match with the modified formula for the reflection coefficient is good.

The paper is organized as follows. We briefly review the physical model used by Silin et. al. in section 2. This is followed by a brief discussion of the modified formula in section 3. In section 4, we discuss the processing methodology followed by results and discussion in section 5.

The Model

The system under study is a homogeneous porous medium filled with a viscous fluid. We consider a p-wave aligned along the x-axis of our coordinate system. From Newton’s second law :

\[ \rho_b \left( \frac{\partial^2 u}{\partial t^2} \right) + \rho_f \left( \frac{\partial W}{\partial t} \right) = \frac{1}{\beta} \left( \frac{\partial^2 u}{\partial x^2} \right) - \frac{\partial p}{\partial x}, \]  

where u is the mean displacement of skeleton grains, W is the superficial or Darcy velocity of fluid measured relative to skeleton, p is the fluid pressure and \( \beta \) is the drained bulk compressibility.

Darcy’s law

\[ W = -\rho_f (\kappa/\eta) \left( \frac{\partial \phi}{\partial x} \right), \]  

(where \( \rho_f \) is the fluid density, \( \kappa \) is permeability of the medium, \( \eta \) is the coefficient of viscosity of the fluid and \( \phi \) is the flow potential) holds for steady state flow. The same must be modified if the flow is transient or there are abrupt changes in the pressure field. Redistribution of pressure field is not instantaneous because fluid flow occurs between pores. Taking \( \tau \) to be the characteristic redistribution time, Darcy’s law is modified to
\( W + \tau (\partial W / \partial t) = -\left(\kappa / \eta\right)(\partial \phi / \partial x), \)  \[ (3) \]

\[ \text{where } d\phi = \left(\frac{dp}{\rho_f}\right) + \left(\frac{\partial^2 u}{\partial t^2}\right)dx \]

Mass conservation equations (or continuity equations) for the fluid and skeleton can be shown to lead to the following equation:

\[ (\partial^2 u / \partial t \partial x) + \beta \phi (\partial p / \partial t) = -\left(\partial W / \partial x\right) \ldots (4) \]

where \( \phi \) is the porosity and \( \beta \) is the fluid compressibility. The above equations can be shown to lead to the famous Biot’s equations of poroelasticity.

The above approach has the advantage that one can find a (low frequency) asymptotic expression for the coefficient of reflection.

Taking plane wave solutions of the form:

\[ u = U_s e^{i(wt-kx)}, W = W_f e^{i(wt-kx)}, p = p_0 e^{i(wt-kx)} \]  \[ \ldots \ldots (5) \]

and substituting in eqns. (1), (3) and (4), it can be shown that \( v = \omega/k \) has two solutions. Thus there are two kinds of P waves in such a medium—a slow and a fast P-wave. Further, the wave number \( k \) and thereby the velocity \( v \) are complex. The imaginary part of \( k \) is negative as it should be for a dissipative medium. The solution obtained by Silin et al. is an asymptotic one i.e. in terms of the parameter \( \varepsilon = (\rho_b \kappa / \eta) \omega \), which is quite small for typical values of \( \rho_b, \kappa, \eta \) and for frequencies less than 1 kHz.

The reflection coefficient for an interface between an elastic medium and a fluid saturated medium is obtained using eqns. (1), (3), (4) and appropriate boundary conditions. The case considered is that of normal incidence. We state below the result obtained for the reflection coefficient \( R \)

\[ R = R_0 + R_1 (1 + i) \varepsilon^{1/2}, \ldots (6) \]

where \( R_0 \) and \( R_1 \) are constants.

### Interference Effects

We incorporate below the effect of tuning into the above formula. For thickness \( h \) less than \( \lambda/4 \), amplitude is roughly proportional to thickness[6]. The important parameter here is \( h/\lambda \). Decreasing thickness or increasing \( \lambda \) amounts to the same thing. For \( h/\lambda \) less than 0.25, amplitude is proportional to thickness for constant \( \lambda \) and proportional to \( 1/\lambda \) for constant \( h \). In the present case, thickness is constant at a given (well) location and \( \lambda \) is changing.

Taking \( v \) to be the interval velocity of the bed, the tuning condition

\[ h/\lambda = 0.25 \]

leads to a tuning frequency of

\[ f = 1/2T \]

where \( h = \nu \Delta t \) and \( T = 2 \Delta t \). Thus for constant thickness \( h \) and \( f < 1/2T \), amplitude is proportional to \( 1/\lambda \) or \( f \). In this range, the formula (6) gets modified to

\[ R = \{R_0 + R_1 (1 + i) (\rho_b \kappa w / \eta)^{1/2}\} k w, \ldots (7) \]

where \( k \) is a constant of proportionality.

By virtue of the above formula the reflected amplitude goes to zero at zero frequency, as it should due to interference effects. However, from formula (6), for usual practical values of parameters, the reflection coefficient falls with frequency and has its maximum at zero frequency.

### The Methodology

Special care needs to be taken in processing to preserve original amplitudes and low frequencies in the data. Preferably, low frequency sensors should be used in acquisition. Stretch effects in NMO spoil the frequency content. Goloshubin et al. suggest target oriented processing where NMO is used only for determining the shifts, which are applied later as static shifts. Migration algorithms must be ‘true amplitude’. We have however eliminated stretch effects, as much as possible, using deep mutes, and worked with Kirchoff migration in this preliminary study. Wavelet transform has been used for isolating different frequency components.

### Results and Discussion

We have taken sections connecting well locations 1 (major gas producer), 2 (dry) and 3 (gas with traces of oil) at different frequencies. Fig. 1 shows the basemap for the area and figs. 2, 3 and 4 show section plots connecting these wells. Fig. 5 shows 10 Hz processed section connecting well 2 and 3. Note the high amplitude at the producing zone at well 3 and no ‘amplitude high’ anywhere at well 2. Fig. 6 shows the same section processed with wavelet transform of 8.24 Hz. Fig. 7 shows a 9 Hz section connecting well 1 (major gas) and well 3 (gas with traces of oil). Note the high amplitudes in the producing zones. In Fig. 8 we show the same section processed at 18.95 Hz. We can clearly see considerable reduction of amplitude in the producing zone at well 1 and almost nil amplitude in the producing zone at well 3. Fig. 9 shows a section connecting well 2 (dry) and well 1 (major gas) at 16.4 Hz.
Fig. 10 shows a plot of the ratio of the instantaneous amplitude at the top of reservoir at well 1 and the input wavelet amplitude at a given frequency (which we call the reflectivity parameter) vs. frequency. The plot shows the fall off of reflection amplitude on either side of peak frequency at 10.88 Hz. This is expected as per formula (7). For comparison, we have also included the power spectrum at well 1 in Fig. 11. This has its peak at 13.49 Hz and is quite different from the reflectivity parameter plot.

Fig. 12 shows a section from another area connecting well 1'(dry) and well 2'(oil) at 5 Hz. We see a clear 'amplitude high' at the producing zone at well 2'. In Fig. 13, we show the same section at 15 Hz which shows an ‘amplitude high’ at well 1’ as well. This is because of coal seams present at that level which shows up now because of better (input) amplitude at this higher frequency.

Although results from this preliminary study are encouraging, further work needs to be done to establish this method. A 3D volume with a number of wells(dry, gas or oil producing) would be an ideal testing ground for the method. In this study, we plan to carry out processing in a manner that is frequency preserving and ‘true amplitude’.

**Conclusions**

In this preliminary study, we have studied the frequency dependent behaviour of reflection amplitudes from seismic data and found ‘amplitude highs’, at low frequencies in producing zones, which disappear at high frequencies. Correlation with well data is good. We have also modified Silin et. al’s formula for the reflected amplitude taking tuning effects into account. Further work, in which proper care of frequency and amplitude issues is taken, on a 3D volume with sufficient well control is envisaged.

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**References**


Fig. 1 Basemap of the area under consideration with locations of well 1, 2 and 3.

Fig. 2 Normal section plot connecting well 2 & 3

Fig. 3 Normal section plot connecting well 1 & 3

Fig. 4 Normal section plot connecting well 1 & 2
Fig. 5 Processed section at 10 Hz connecting well 2 (dry) & well 3 (gas with traces of oil). The circled area is the producing zone at well 3.

Fig. 6 The above section at freq. 8.24 Hz.

Fig. 7 Processed section at 9 Hz connecting well 1 (major gas) and well 3 (gas with traces of oil).

Fig. 8 The above section at 18.95 Hz. Note the considerable reduction of amplitude at well 1 and almost zero amplitude at well 3.
Fig. 9  Processed section (16.4 Hz) well 2 (dry) & well 1 (major gas)

Fig. 10  Reflectivity parameter i.e. the ratio of the instantaneous amplitude to the input amplitude at a particular frequency vs. frequency at well 1.

Fig. 11  Power Spectrum at Well1 shows (primary) peak at 13.49 Hz.

Fig. 12  Section from another area connecting well 1'(dry) and well 2'(oil) -- transform at 5 Hz.
Fig. 13  Same section as in fig. 12 at 15 Hz.