Stable optimization-based deghosting for broadband seismic data processing

Denisov M.S.*, GEOLAB, Firssov A.E., GEOLAB
denisovms@gmail.com

Keywords
Seismic ghost reflection, optimization, inversion, convolutional model

Summary
A method to eliminate a seismic ghost reflection is presented. It allows deghosting from both the source- and receiver- sides. An estimator of the reflection coefficient and ghost time delay is incorporated into the deghosting scheme.

Introduction
Ghost reflections occur in the process of data recording in marine seismic surveys. Each wavelet reflected from the subsurface interfaces is accompanied by two ghosts: the source- and the receiver- side ones. Moreover, on the receiver side, the upgoing source-side ghost produces another event: a higher-order ghost reflected from the sea surface both on the source- and receiver- sides. Imagine the airgun producing a delta pulse wavelet. Then the recorded wavelet will consist of four spikes. This distorts the seismic signal and degrades data resolution. Due to the ghost reflection, the amplitude spectrum will suffer from the lack of low frequencies, and in some cases the notch filtering effect might occur. Special tools for ghost elimination should be applied to enable broadband processing. Although seismic deghosting is a long-standing problem, it still remains challenging objective.

Many publications trying to tackle this problem are known. Very often, the proposed solutions require data registration with making complementary measurements (see e.g., Barr and Sanders 1989), but since recording using single-component flat streamers is most commonly used in practice, here we consider the pressure-only data and multicomponent deghosting is beyond the scope of our research. This also holds for slanted streamers (Soubaras and Dowle 2010), random-depth sources (Carlson et al. 2007), etc.; our aim is to develop a processing-based, rather than an acquisition-based solution.

The conventional manner to arrive at a deghosting scheme is to write an expression that describes the ghost simulation operator, and then perform the inversion of this operator. The resulting filter is applied to the acquired data. The approaches most extensively developed in the literature imply 2D operators that account for the wave emergence angle. A group of receiver- side deghosting algorithms that imply prediction and subsequent subtraction of the ghost are known and are being discussed extensively. They use either the Kirchhoff integral (Beasley et al. 2013) or a shift in the tau-p domain (see e.g., Masoomzadeh et al. 2013, Wang et al. 2013) for prediction. Hence, the model of a ghost reflection is computed by means of 2D convolution. Some other methods utilize 2D Wiener filtering theory to compute the deghosting operator. Here we intentionally ignore these approaches since our experience proves that good performance of such filters can be expected only in case of fine spatial sampling. In other words, the spacing between receivers should be small; otherwise the result will contain strong alias noise. The receiver interval suitable for the application of these techniques depends on the receiver depth, but, as it follows from our calculations, in most practically important cases it should be less than 5m. The approach proposed here applies a single channel ghost prediction. Its advantages over the 2D schemes are similar to the advantages of the well-known static shift correction (Yilmaz 1986) over the redatuming (Bevc 1997) methods in the problem of compensation for the near-surface effects in land seismic data processing. On one hand, in case the near-surface velocity-depth model in known exactly, the perfect solution for the problem of its correction (that implies sinking of the sources and receivers to a new datum located below the near-surface), will be provided via the redatuming approach. The data transformation operator is derived from the Kirchhoff integral. On the other hand, such solution requires a perfect data acquisition pattern. Particularly, very small spacing between both the sources and the receivers is required. Otherwise, the application of the operator that has been derived from the integral expressions and, hence, implies continuous rather than discrete recorded wavefields can produce strong alias artifacts. Moreover, ghost prediction needs wavefield extrapolation to small distances, not more than approx. 20-30m. This requires the application of a so-called near-field rather than the conventional asymptotic far-field operator. The former is unstable particularly because of the division by the extrapolation distance squared. As it is known from the land data processing experience, in most practically important cases the correction for the near surface effects using the static shifts shows better performance than redatuming.

Another reason why we ignore the 2D operators in this study, is their inability to perform the source-side deghosting. Wavefield extrapolation operator is always applied to a recorded wavefront, and the receiver-side ghost prediction is applied to the CS gathers that contain this wavefront. The towed streamer marine acquisition is end-on-spread, so a common receiver (CR) gather that, based on the seismic reciprocity principle, might be considered as the one containing the source-side wavefront, cannot be formed.

All the known methods require the values of the free surface reflection coefficient and the ghost time delay. The conventional choice for the coefficient is -1, although Hatton (2007) shows that in many cases this
assumption is violated. We apply an optimization-based approach that allows us to incorporate an estimator of the reflection coefficient and time delay directly into the deghosting scheme. Although some attempts to obtain the estimates of the reflection coefficient and the receiver (or source) depth are known (see e.g., Perz and Masoomzadeh 2014), the estimator proposed works as the conventional minimum-phase spiking deconvolution filter rather than the ghost model-based recursive filter containing the two unknown parameters. The statistical deconvolution operator estimates the ghost amplitude and time delay in an implicit manner. It inevitably provides a biased estimate because of its finite impulse response nature, as the true ghost model-based operator is a recursive one, i.e. it has an infinite impulse response. What is more, the possibility of using the minimum prediction error dispersion criterion needs a scrupulous justification.

Model

We use the statistical convolutional model of a seismic trace (Robinson 1985), \( z(t) \), that can be presented as

\[
z(t) = r(t)^a p(t),
\]

where \( t \) is time, \( r(t) \) is the sequence of the reflection coefficients, \( p(t) \) is a seismic wavelet, asterisk stands for convolution. The additive noise factor that is usually put into the right-hand side of equation (1) is dropped here for the simplicity of the subsequent discussion. After the geometrical spreading correction, \( r(t) \) is assumed to be stationary white noise, the so-called independent identically distributed (i.i.d.) process.

We shall call the wavelet excited by the airgun – the source wavelet. We shall call the recorded wavelet, which is a sum of the source wavelet and the ghost – the complex wavelet. Let \( w(t) \) stand for the source wavelet. Then the complex wavelet containing the receiver-side ghost will be

\[
g(t) = w(t) + a_r w(t + \tau_r),
\]

where \( a_r \) is the receiver-side reflection coefficient, \( \tau_r \) - receiver-side delay of the ghost reflection. The complex wavelet containing both the receiver- and source-side ghosts will be

\[
p(t) = g(t) + a_s g(t + \tau_s).
\]

Having substituted the equation (3) into (2) we arrive at

\[
p(t) = w(t) + a_r w(t + \tau_r) + a_r w(t + \tau_r) + a_r a_s w(t + \tau_r + \tau_s).
\]

The usual choice made in most known deghosting algorithms is \( a_r = -1 \) and \( a_s = -1 \). This inevitably leads to the instability of the deghosting procedure. In our approach, we do not make this supposition, but apply an estimator for both the source- and receiver-side reflection coefficients. Due to such principle, our scheme turns out to be stable, no increase of energy is allowed. The geophysical reason for deviation of the reflection coefficient from -1 is given by Hatton (2007). Based on the physical modelling experiment, he demonstrated that \( a_r \) might be larger than -1 (clearly, this may also hold for \( a_s \).)

The approximate values of the source- and receiver-side delays are assumed to be known. They will be refined during our deghosting procedure to obtain the optimal result.

We suppose that the wavelet is causal (\( w(t) = 0 \) for \( t < 0 \) ) with normalization \( w(0) = 1 \).

The expression for the dispersion, \( \sigma^2 \), of the process (1) is as follows (Ottes and Enochson 1972)

\[
\sigma^2 = E[r^2](1 + \sum_{i=1}^{N} w^2(i)).
\]

For the normalized causal wavelet

\[
\sigma^2 = E[r^2](1 + \sum_{i=1}^{N} w^2(i)).
\]

As it can be easily seen from the last expression \( \sigma^2 \geq E[r^2] \) and reaches its minimum in case of a delta-function wavelet \( w(t) = 0 \) for \( t > 0 \).

**Algorithm**

Here we derive the optimization scheme to obtain the estimates of the source- and receiver-side reflection coefficients and time delays that are the sought-for values. We suppose that the energy of the source wavelet rather rapidly decreases with time: \( w^2(t) \rightarrow 0 \) with \( t \rightarrow \infty \). Such property holds in most practically important cases in marine data acquisition. Hence, the appearance of a ghost reflection leads to the increase in energy of the complex wavelet. In other words,

\[
Q_{SW} < Q_{SW+RG} < Q_{SW+RG+SG}.
\]

where \( Q_{SW} \) is the energy of the source wavelet, \( Q_{SW+RG} \) is the energy of the complex wavelet containing the receiver-side ghost, \( Q_{SW+RG+SG} \) is the energy of the complex wavelet containing both the source- and receiver-side ghosts.

As it follows from expression (4), the dispersion of the deghosted trace will be smaller than the dispersion of the recorded trace. Using similar notations for dispersions we arrive at

\[
\sigma_{SW}^2 < \sigma_{SW+RG}^2 < \sigma_{SW+RG+SG}^2.
\]

Based on equations (1) and (2) we obtain a frequency-domain expression for a seismic trace, \( Z(\omega) \) with a receiver-side ghost

\[
Z(\omega) = R(\omega)W(\omega)(1 + a_r \exp(j \omega \tau_r)),
\]

where \( R(\omega) \) is the reflectivity response spectrum, \( W(\omega) \) is the source wavelet spectrum, \( j = \sqrt{-1} \). An estimate of dispersion is obtained by summing the amplitude spectrum

\[
\sigma^2 = \sum_{\omega=0}^{\omega_{max}} |Z(\omega)|^2.
\]
where \( \omega \in (\omega_1, \omega_2) \) is the signal band. Since the minimal dispersion is achieved for a deghosted trace, the least squares (LS) objective

\[
J(\bar{a}_r, \bar{s}_r) = \sum_{\omega=\omega_1}^{\omega_2} \frac{Z(\omega)}{(1 + \bar{a}_r \exp(\jmath \omega \tau_r))(1 + \bar{s}_r \exp(-\jmath \omega \tau_s))} \]

reaches its minimum on the sought-for parameters. Another useful formulation of this optimization problem is

\[
a_r, \tau_r = \arg \min_{a_r, \tau_r} \sum_{\omega=\omega_1}^{\omega_2} \frac{Z(\omega)}{1 + a_r^2 + 2a_r \cos \omega \tau_r + \varepsilon^2} \quad (5)
\]

Regularization factors might be added to the objective (5).

\[
a_r, \tau_r = \arg \min_{a_r, \tau_r} \sum_{\omega=\omega_1}^{\omega_2} \frac{|Z(\omega)|^2 + \gamma^2}{1 + a_r^2 + 2a_r \cos \omega \tau_r + \varepsilon^2} \quad (6)
\]

In case we use regularization \( \gamma^2 \) in numerator, it will have the sense of the white-noise factor well-known from seismic deconvolution theory. In case we use regularization \( \varepsilon^2 \) in denominator, it will prevent from the instability of division by small spectrum components. Since the objective (6) is non-linear with respect to the unknowns, we use the coordinate descent optimization method. The starting value for the time delay is computed from the depth of the receiver given by the user, which can vary from one gather to another. Having fixed the time delay, we minimize the LS objective (6) with respect to the reflection coefficient and obtain its optimal value. At the next step, we fix this optimal coefficient and run a single-parameter optimization to find the optimal time delay. After several iterations, the process will converge. A useful feature is to introduce a constraint that will limit the area of possible time delays. The user might specify an a priori depth of the receiver and define its uncertainty by setting a range within which this receiver is expected to be.

Having obtained the estimates of the reflection coefficient and time delay, a deghosting filter might be applied according to the expression as follows

\[
Z(\omega) = \frac{Z(\omega)(1 + a_r \exp(\jmath \omega \tau_r))}{1 + a_r^2 + 2a_r \cos \omega \tau_r + \mu^2}, \quad (7)
\]

where \( \mu^2 \) is a regularization parameter. The time-domain equivalent of transformation (7) means recursive filtering (or convolution with an infinite impulse response filter).

An objective similar to (6) is formed and a similar coordinate descent optimization scheme is applied to perform the source-side deghosting. \( Z(\omega) \) in (6) is substituted by the spectrum of the receiver-side deghosted trace \( \hat{Z}(\omega) \) computed using the expression (7). Since the reflection coefficient and the time delay are the same for all the traces within a common source (CS) gather, we apply averaging by summing the objectives (6) and obtain a single coefficient and a single shift for the whole gather. This property explains the statistical stability of the obtained estimates of the sought-for source-side ghost parameters. As in case of receiver-side deghosting, the user might either specify a range of depths for the source or simply fix its depth and perform a single-parameter LS optimization using the objective (6).

As it follows from the minimum energy assumption and the recursive nature of the filter, deghostings from the receiver- and source-side might be done independently. Therefore, the simultaneous estimation of the four unknowns (two reflection coefficients and two time delays) which is a complicated procedure because of the mentioned above non-linearity of the objective, is not necessary. Receiver- and source-side deghostings can be done successively. Output from the first stage is the input for the second one, \( \hat{Z}(\omega) \). It is easy to understand that in the first stage the scheme will automatically detect and, hence, obtain the estimates that correspond to the stronger ghost reflection. After filtering (7) optimization (6) applied to \( \hat{Z}(\omega) \) will estimate the weaker ghost and subsequently eliminate it. In other words, the LS criterion works correctly for both source- and receiver-side parameter estimators applied separately. We expect that in case the reflection coefficients are close \((a_r \approx a_s)\) and the depths of the receivers within a streamer differ, the estimator (6) applied to a CS gather will provide the values of \( a_r \) and \( \tau_r \). After the source-side deghosting we apply the same estimator to the resulting traces in a single-trace manner to obtain the receiver-side values \( a_s \) and \( \tau_s \) with the subsequent deghosting filtering.

**Examples: synthetic data**

We study the performance of the suggested algorithm on a synthetic dataset. First, we add a ghost reflection from only one side (source or receiver). Figure 1 (left panel) shows the synthetic traces with a ghost reflection added to the source delta function wavelet. The ghost amplitude and time delay were the unknowns for our optimization-based deghosting scheme. The result of the unknown parameter estimation and subsequent deghosting with recursive filtering is given in the panel on the right. The ghost reflection has been suppressed almost perfectly. As it follows from the amplitude spectra given in the bottom panels of Figure 1, the signal notch frequencies have been restored.

As it can be easily understood, adding a ghost reflection to a source wavelet acts similarly to its differentiation. Therefore, the shape of the spectrum in the low frequency region is expected to be linear with zero amplitude at zero frequency. As it follows from Figure 1, deghosting has successfully eliminated such linear behavior transforming the spectrum into a broadband one.
A synthetic data example of deghosting from both the source- and receiver-side is presented in Figure 2. The left panel shows the synthetic traces with ghosts. The result of parameter estimation and subsequent deghosting is given in the panel on the right. Again, the ghost reflection has been successfully suppressed. The notch frequencies have been restored.

Similarly to one-side ghost, adding of two ghost reflections from the source- and receiver-sides to a source wavelet acts similarly to its second order differentiation. Therefore, the shape of the spectrum in the low frequency region is expected to be parabolic with zero amplitude at zero frequency. As it follows from Figure 2, deghosting has successfully eliminated such parabolic behavior transforming the spectrum into a broadband one.

**Examples: real data**

Figure 3 gives a real data example. A segment of a raw CMP gather (left panel) and the result of deghosting (right panel) are shown. Estimates of the wavelet amplitude spectra are presented in the Figure 4: raw data spectrum (left panel) and spectrum after deghosting (right panel). The optimization-based approach described above was applied.

As it can be seen from Figure 4, deghosting has broadened the amplitude spectrum of the wavelet and has added both the low and high frequencies.

Figure 5 gives another marine real data example. Again, a segment of a raw CMP gather (left panel) and the result of deghosting (right panel) are shown. The estimates of the wavelet amplitude spectra are presented in Figure 6: raw data spectrum (left panel) and spectrum after deghosting (right panel). The optimization-based approach described above was applied. As it can be seen from Figure 6, deghosting has flattened the spectrum, adding the low and high frequencies. Besides, the energy in the vicinity of a notch frequency (approx. 55Hz) has been well restored.
Conclusions

We have proposed an optimization-based seismic deghosting scheme for broadband data processing. One of its main advantages is the possibility of automatic adjustment of the value of the unknown reflection coefficients on the source- and receiver-sides. Usually, the reflection coefficient that equals -1 is used. Physical modeling results prove that the ghost is very often characterized by an “efficient” reflection coefficient that can significantly differ from -1. Our method allows data processing with ghosts characterized by an arbitrary reflection coefficient. Moreover, the optimization provides the possibility of refinement of the depth of the source and the receiver. Source- and receiver- ghost time delays might be different.

With our method, ghost elimination can be done from both the source- and receiver- sides. To improve the statistical reliability of the estimates obtained, the source- side ghost parameters (time delay and amplitude) are averaged within a CS gather. We apply a single-channel rather than a multichannel deghosting. Although this approach has some limitations, its results are stable and expectable. In most practically important cases (especially for shallow streamers) it is preferable over its multichannel (2D) analog. It produces no alias noise and allows deghosting from both the source- and receiver- sides.

The LS objective we have formed is based on the assumption of a rapidly decaying source wavelet. Although this holds true for most cases, our experience proves that application of resampling (transformation that moves the top frequency of the signal band to the Nyquist frequency and the low frequency of the signal band to the zero frequency) compresses the wavelet and, hence, improves the performance of our optimization-based scheme. After deghosting we apply inverse resampling to obtain a trace with the original signal frequency band.

$L_p$ norm can be considered as an alternative to LS norm. Its statistical properties are different from the LS norm. It optimizes the data resolution based on the sparse spike requirement. Since, as mentioned above, after deghosting four delta functions that form a complex wavelet are replaced by a single delta function source wavelet, we expect this approach to also be promising and our future research will be partly devoted to this topic.

The synthetic and real data examples given in the abstract show that the method presented is promising and can be a useful deghosting tool for a processing geophysicist.

References


Beasley, C., Coates, R. and Ji, Y., 2013, Wave equation receiver deghosting. 75th EAGE meeting. Expanded abstracts.


Yilmaz, Oz, 1986, Seismic Data Processing. SEG.