Seismic modeling and reverse time migration on a high performance computing System

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Summary

Seismic modeling and reverse time migration based upon the solution of two-way acoustic wave equation are two important applications in the oil & gas industry. Despite showing promising imaging capabilities, it has a limited use in the industry due to high computational cost. In this paper we propose the implementation of modeling and reverse time migration algorithms on India’s largest high performance computing system, called EKA. Solving acoustic wave equation accurately in complex media is the key to these algorithms. The wave propagation problem is formulated as a hyperbolic system of equations and is solved on a regular mesh using a finite difference method which is second order accurate in time and fourth order accurate in space. The performance is achieved by parallel implementation using domain decomposition, on a large compute cluster. Scalability and efficiency is achieved by proper structuring of the codes.

Keywords: Seismic Modeling, Reverse Time Migration, Wave Equation, High Performance Computing.

Introduction

The extraordinary challenge that the oil and gas industry faces for hydrocarbon exploration requires the development of leading edge technologies to recover an accurate representation of the subsurface. Seismic modeling and reverse time migration (RTM) are areas of significant research in the industry. Wave equation modeling is useful for understanding complex imaging problems, for processing and algorithm testing, and for AVO analysis, whereas RTM is a state of the art depth migration technique for imaging subsurface geological structures from the recorded seismic data (Baysal et al., 1983; McMechan, 1983; Yoon et al., 2003; Farmer et al., 2006). Both of these techniques are based upon the solution of full acoustic wave equation. Unfortunately they are highly compute intensive. Advances in high performance computing technologies resulted in renewed attention from the seismic community to these techniques.

In the last two decades the power of single CPU has increased by several orders of magnitude. But applications like Seismic modeling and RTM need computational resources far greater than a sequential computer can provide. Parallel processing has proven to be a viable solution to improve performance, which uses the power of multiple computers connected together by high-speed network.

In this paper we will discuss how seismic modeling and RTM can take advantage of the massively parallel computing infrastructure. First the wave propagation methodology used for both modeling and RTM is discussed. Then implementation of the algorithms in a parallel computing environment is shown along with the scalability and performance issues.

Wave propagation in heterogeneous media

The mathematical model for acoustic wave propagation in 3D heterogeneous media consists of coupled second order partial differential equations governing motions in x, y and z directions. Instead of solving second order coupled partial differential equations we formulate them as a first order hyperbolic system (Virieux 1986, Vafidis 1988, Dai et al. 1996):

$$\frac{\partial^2 Q}{\partial t^2} = A \frac{\partial Q}{\partial x} + B \frac{\partial Q}{\partial y} + C \frac{\partial Q}{\partial z}$$
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Here Q is a vector comprising of particle velocities and stress components and A, B, C are matrices containing physical parameters of the medium. They are given by

\[
Q = \begin{bmatrix} \rho & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \rho \end{bmatrix}, \quad A = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

where \(p\) is the negative pressure wavefield, \(K\) is the incompressibility, \(\rho\) is the density and \(u, v, w\) are partial velocities in \(x, y, z\) directions respectively. This formulation does not contain any derivatives of physical parameters. Thus, we need not calculate the gradients of physical parameters that may cause singularity in the numerical solution due to sharp changes in the physical properties.

The solution to the above partial differential equations is obtained by using a finite difference approach. For this purpose the model is discretized into a number of grid points and the wavefield is calculated at each grid point as a function of time. An explicit finite difference method based on the MacCormack scheme is used for the numerical solution (Mitchell and Griffiths, 1981). This scheme is fourth order accurate in space and second order accurate in time. The model discretization is based upon regular grid. Sponge boundary conditions are used for attenuating the reflected energy from the left, right and bottom edges of the model (Sochaki et al. 1987). A free-surface boundary condition or a sponge boundary condition is used for the top edge.

Seismic Modeling

The basic problem in theoretical seismology is to determine the wave response of a given model to the excitation of an impulse source by solving the wave equation. Numerical seismic modeling aims at simulating seismic wave propagation in a geological medium in order to generate synthetic seismograms. Among the numerous approaches to seismic modeling, direct methods based upon approximating the geological model by a numerical mesh are of particular interest. This approach can give very accurate results, however it requires large computational resources.

Reverse Time Migration

RTM makes use of the solution of hyperbolic system of equations in the following manner. The algorithm is implemented in shot gather domain. First the forward extrapolation of the source wavefield is carried out for each shot location through the gridded velocity model using finite difference method. The wavefield must be stored at each time step for application of imaging condition. Next the recorded wavefield (shot gathers) is backward propagated in time and is stored at each time step. Forward propagated wavefield and the reverse extrapolated wavefield are cross correlated to obtain the image. RTM can handle complex velocity models and is capable of imaging steep dips, turning waves, horizontally travelling waves, prism waves, ghosts and intrabed multiples (Jones et al., 2006).

Parallel Implementation

The most important part of parallel programming is to map out a particular problem on a multiprocessor environment. The problem must be broken down into a set of tasks that can be solved concurrently. The choice of an approach to the problem decomposition depends on the computational scheme. In this study we have used MacCormack scheme for finite differencing which is fourth order accurate in space and second order accurate in time. For this scheme the calculation of the wavefield at a grid point of the advanced time level involves the knowledge of the wavefield at thirteen grid points of the current time level. Therefore, it is a thirteen point differencing star. Therefore, if we use a domain decomposition scheme for solving this problem second order neighbors will be involved in communication. Domain decomposition involves assigning subdomains of the computational domain to different processors and solving the equations for each subdomain concurrently. A checkerboard partitioning is applied here.

In checkerboard partitioning, domain is divided in all three directions creating smaller subdomains. In uniform checkerboard partitioning, all subdomains are of the same size. These subdomains have to be distributed among processors and no processor gets the complete plane. In order to calculate the wavefield at the grid points of the subdomain, at each time step, the required boundary grid points should be interchanged between the processors. In
the case of checkerboard partitioning the ghost point communication is in all three directions, i.e. each processor should exchange the boundary grid points with its six neighbors.

We have used MPI for the parallel implementation on a cluster of Nodes. Each node comprises of several cores. To setup the domain decomposition, we have used Cartesian topology approach of MPI. This helps in identifying the neighbors for the inter-processor communications during exchange of the wavefield along the subdomain boundaries. MPI Send-Receive function is used for this purpose. For all other parameter distributions usual MPI send, receive and broadcast functions have been used.

**Performance Analysis**

We performed the benchmark tests of the parallel 3D acoustic wave modeling and RTM algorithms for several problem sizes on EKA, CRL’s high performance computing system (HPC). EKA is a HPC system with 1800 compute nodes, where each node is a dual quadcore processor. The nodes are connected by 20 GBps infiniband interconnect with less than 2µs latency. The peak and sustained performance of the system is 172 TFlops and 132.8 TFlops respectively.

The accuracy and efficiency of acoustic wave modeling was tested by applying it to Marmousi-II model (Martin, Wiley and Marfurt, 2006). Originally this model is a 2D model, but we extended it in the third dimension by replicating the 2D model. The purpose was to test and benchmark the algorithms. Figure 1, shows the x-z slice of this complex model, where x denotes the horizontal axis and z denotes the vertical axis. Different colours represent different p-wave velocities in the model.

In this paper we show the results of 3D acoustic wave modeling for a problem size of 1400 X 400 X 800. For the model shown in Figure 1, ∆x & ∆y are 6.25m and ∆z is 5.0m. The time step is 0.5msec. The source is at 1.25km and the data is recorded by 580 receivers located from 1.35km to 8.6km. The nearest offset is 100m and the farthest offset is 7250m. 8000 time steps are required in order to produce synthetic data for 4.0sec. Figure 2 shows the synthetic seismograms obtained by 3D acoustic wave modeling algorithm for Marmousi-II model. Since the model is quite complex we do not explicitly see any

intrabed multiples, although they are also present in the seismograms.

![Figure 1: A schematic of Marmousi-II velocity model. The colour bar on left represents p-wave seismic velocities in m/sec.](image1)

![Figure 2: Synthetic seismograms obtained from 3D acoustic wave modeling algorithm for the Marmousi-II model.](image2)

Tables 1 shows the performance of the acoustic wave propagation algorithm. A minimum of 32 cores were needed to run for the size of the model chosen for this study. Therefore all the other runs are compared with 32 core run. The compute times are compared for 1000 time steps only. As the number of cores increase from 32 to 512, a sixteen fold increase in the number of cores, the compute time for acoustic wave modeling reduces by a factor of 10, approximately 64% efficiency. From the table 1, one can easily observe that a large amount of compute time is necessary to produce one shot gather.
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The performance of the parallel 3D RTM algorithm was tested using SEG/EAGE overthrust model shown in Figure 3. The algorithm was tested for a model size of 801X193X378 with grid spacing of 12.5m in all directions. A 15 Hz Ricker wavelet, time step of 1.0ms and the source and receiver spacing of 25m was used for migrating each shot gather. The performance of the algorithm is shown in Table 2 for a single shot gather. Hundreds of shot gathers were migrated and then stacked to obtain the final migrated image shown in Figure 4.

For the performance analysis four cores on each node were used. We could not run the shot gather with less than 4 nodes for this problem size, because the memory on each node was not adequate. From the table one can easily see that the speedup is close to linear. Further optimization of this code is currently in progress, and the results will be shown during presentation.

![Figure 3: A schematic of the SEG/EAGE overthrust model. Colour bar represents the p-wave seismic velocity is m/sec.](image)

![Figure 4: The seismic image obtained after 3D RTM of the overthrust model along two inlines. The velocity model is also superimposed on the migrated section.](image)

### Table 1: Performance of Acoustic wave modelling algorithm as a function of number of cores.

<table>
<thead>
<tr>
<th>No of cores</th>
<th>Time for 1000 time steps (sec.)</th>
<th>Relative speedup over 32 cores</th>
<th>Relative efficiency over 32 cores</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>6240</td>
<td>1.0</td>
<td>100.0%</td>
</tr>
<tr>
<td>64</td>
<td>3323</td>
<td>1.88</td>
<td>94.00%</td>
</tr>
<tr>
<td>128</td>
<td>1799</td>
<td>3.47</td>
<td>86.75%</td>
</tr>
<tr>
<td>256</td>
<td>1036</td>
<td>6.02</td>
<td>75.25%</td>
</tr>
<tr>
<td>512</td>
<td>606</td>
<td>10.3</td>
<td>64.37%</td>
</tr>
</tbody>
</table>

### Table 2: Performance of 3D RTM algorithm for a single shot gather as a function of number of nodes. Four cores on each node were used for computations.

<table>
<thead>
<tr>
<th>Number of Nodes</th>
<th>Model Size</th>
<th>Speedup over 4 nodes</th>
<th>Time Step</th>
<th>Run-Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>801X193X378</td>
<td>1.0</td>
<td>1000</td>
<td>982.33</td>
</tr>
<tr>
<td>8</td>
<td>801X193X378</td>
<td>1.85</td>
<td>1000</td>
<td>529</td>
</tr>
<tr>
<td>16</td>
<td>801X193X378</td>
<td>3.67</td>
<td>1000</td>
<td>267.73</td>
</tr>
<tr>
<td>32</td>
<td>801X193X378</td>
<td>7.08</td>
<td>1000</td>
<td>138.66</td>
</tr>
</tbody>
</table>

### Conclusions

In this paper we have shown the implementation of 3D acoustic wave modeling and reverse time migration algorithms on a high performance computing called EKA. This machine is India’s largest high performance computing system. The 3D acoustic wave propagation problem is formulated as a system of hyperbolic system of equations. A domain decomposition scheme with MPI message passing libraries, has been used for parallel implementation of these codes. The performance analysis of these codes was carried out on a large compute cluster. The results demonstrate that a near linear speed up can be achieved by proper structuring of the codes.
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References


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