prescribed over a circular boundary. of radius a, its value f_0, at the centre of the circle is given by the arithmetic mean value theorem

$$f_0 = \frac{1}{2pa}\text{ò}_c f(q)dq, \tag{5}$$

where q defines a boundary point and dq defines the arc element at q. Now let us divide the circular boundary c into n equal subinterval Dc_j, j=1,2 ....n and assume that f is constant over a subinterval Dc_j,its value being associated with the nodal point of Dc_j. Under this assumption, the expression (5) leads to

$$\frac{2paf_0}{Dc} = \overset{n}{\underset{j=1}{\text{å}}} f_j = \frac{2pa}{n}\underset{j=1}{\text{å}} f_j$$

$$\text{or } n f_0 = \overset{n}{\underset{j=1}{\text{å}}} f_j \tag{6}$$

Where f_j is the constant value of f over the jth subinterval and Dc is the length of a subinterval.

The above formula is valid for a large value of n so that the variation of f over a subinterval is negligibly small. The 4-point formula is a particular case of (6) for n=4, implying that f is constant over a quarter of the circumference of the circle of radius i.e f is constant over the large interval $Dc_j = pa/2$. Since the potential field has a high gradient near the causative mass, the 4-point formula is likely to fail near the causative mass. It is approximately valid for the circle lying far off the causative mass, and it gradually distorts with the increase in gradient of the field as the circle approaches the causative mass.

Further,as the continuation is carried out in successive downward steps, the error accumulated in the computed f_j at a shallower step becomes multiplied in the next downward steps. Thus the function value obtained at a depth by taking several downward steps may not be of any practical use. It is our experience that for the input data with less than 1% random error the computed field oscillates either at 3rd or 4th step of downward continuation, irrespective of spacing of the data over the datum line. This conclusion finds its support in almost al the examples furnished by Roy (1966)

Though the formula fails to work, some credit must go to Roy for pointing out the downward continuation of observed data leads to finding of depth to the causative mass.

2.5 A mis-concept on origin of 4-point formula

$$\tilde{g}(r) = \frac{1}{n}\overset{n}{\underset{j=1}{\text{å}}} g_j(r, q), \tag{7}$$

The 4-point formula of Roy (1966) is described in Applied Geophysics (Telford et al 1975, p. 84) as a modification of the empirical gridding formula of Griffin (1949). Griffins formula states that

Where $g_j(r, q)$ are the gravity values on the circumference of a circle that lies on a horizontal plane, g(r) is the regional gravity value at the centre of the circle and r is the radius of the circle.

Griffin's formula is an empirical one that uses three-dimensional gravity data specified over a horizontal plane, say (x,y) plane of a cartesian reference frame with z-axis upward. The 4-point formula of Roy (1966), on the other hand, uses the two dimensional gravity field defined in (x,z)-plane. Roy (1966) derived the formula by extrapolation of the field in downward direction considering the first three terms of Taylor's series and subsequent use of the Laplacian property $¶^2g/¶z^2 = -¶^2g/¶x^2$. The two formulae look alike but they are distinctly different in their derivation and application. As such, it is wrongly quoted in Telford et al (1975) that the 4-point formula of Roy (1966) is a modification of Griffin's empirical formula.

3. Significance of Roy's work

(i) When it was apparently accepted, following Skeels (1947), that a unique depth to the causative mass cannot be determined from its potential response, Roy (1962) theoretically arrived at the conclusion that a unique depth to the causative mass can be obtained if the geometry of the subsurface causative mass is known and its density (contrast) is assumed a constant.

Hammer (1977) demonstrated its successful application to field data by determining depth to the basement at isolated points in a basin.

(ii) The very concept of Roy (1966) that ''downward continuation of observed potential data widely oscillates on reaching the target'' is a turning point in the philosophy of depth determination from observed potential data. For the first time, after 1947, it was pointed out by Roy that a unique depth to the causative mass can be determined from its potential response alone. Though the formula he presented failed to work because of the approximations involved in it, his conclusion in 1966 reopened the door after a gap of nearly 20 years for further work on depth-determination. Application of the concept however was not immediately accepted by the geoscientific community probably because of the absence of an elaborate theoretical discussion on it and also probably the difficulties faced in achieving it numerically.

The conclusion of Roy (1966) has now been theoretically established (Laskar 1980, 2000) and a numerically effective technique has been developed for finding depth to the causative mass by downward continuation of observed potential data along a vertical passing through the causative mass (Laskar & Singh 1993, Laskar et al 1996). The technique has been successfully tested on field data of various basins in India (Laskar 1991, Laskar et al 1996).

4. Conclusion

(i) When it was assumed, following Skeels (1947), that the depth to a subsurface causative mass cannot be uniquely determined from its potential response alone. Roy (1962) came out with a theortical conclusion that it can be obtained uniquely if the geometry of the causative mass is known and its density (contrast) is assumed constant. Application of the idea has been successfully demonstrated by Hammer (1977) on field data and it is now being used as a routine procedure by the industries to find spot-depths in a basin.

(ii) The demonstration of Roy (1966) that downward continuatoion of observed potential data oscillates widely on reaching the target can be treated as a turning point in the concept of depth determination from observed potential data. This reopened a nearly closed chapter on depth determination, and this finally culminated in development of a successful technique on determination of depth by downward continuation of observed potential data along a vertical passing through the causative mass.

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Stratigraphic Inversion For Enhancing Vertical Resolution

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Summary:

Seismic inversion combines geophysical, geological and petrophysical data through a robust inversion scheme to extract the more reliable stratigraphic information and reservoir properties from the seismic data. The benefits of acoustic impedance obtained after inversion are well established for stratigraphic interpretation in comparison to conventional seismic data analysis and are outlined. Post stack seismic inversion techniques work mostly on the basis of one dimensional convolution model of seismic trace, which forms the basis for inversion. In order to provide pre-inversion checks, possible measures to ensure the validity of assumptions and the compatibility of the data with velocity/impedance constraints, a complete methodology incorporating all the steps of practical post stack inversion is described in detail. In this study an attempt has been made to demonstrate the application of this complete inversion sequence using model based inversion scheme by showing four examples. First example shows that synthetic seismic response generated from the wedge shaped geological model is affected by tuning and interference of side lobes and generate internal false geometry. The inverted acoustic impedance is free of tuning, side lobe interference and is very close to real geological model. Second example shows that gradational variations of impedance contrast are not possible to visualize from the seismic data but they can be identified from the inverted impedance section. Third example shows that acoustic impedance display provides much finer layer by layer information in contrast with seismic data and it bridges the gap between vertical resolution of well log measurements and seismic data, simplifies the lithologic and stratigraphic definitions of the sub-surface. Fourth example demonstrate the effectiveness of acoustic impedance over conventional seismic in delineation of thin sand reservoirs which are recorded as composite response in seismic data.

Introduction:

Stratigraphic interpretation of 3-D seismic data is usually performed on a migrated volume having limited signal bandwidth. The low frequency components in seismic data are limited by natural frequency of receivers below which signal gets distorted. The various processes in earth eat up the high frequency components and fix the highest corner frequency, which can be achieved under ideal conditions of acquisition and processing. This band-limited nature of propagating seismic wavelet fixes the practical limit of seismic resolution and seismic record contains information only on that part of the spectrum of the stratified earth, which corresponds to the wavelet's bandwidth. While correlating reflectors in migrated data volume, geophysicists interpret interface geometry, which depends upon the variation of impedance contrasts and thin layer interferences. In spite of phenomenal growth of interest in the use of seismic attributes derived from 3-D seismic volumes to predict subsurface properties, most often, it has been observed that seismic attributes derived from time, amplitude and frequency do not provide adequate reservoir properties on a layer-by-layer basis. Layer-by-layer information can be derived by means of stratigraphic inversion of post stack and pre-stack seismic data in terms of acoustic impedance. While carrying out stratigraphic inversion, attempts are also being made to restore the lost low as well as high frequencies. Figure (1) summarizes the different types of stratigraphic inversions used for inverting post stack and pre-stack seismic data in the industry. The inverted results provide an edge over normal seismic data (amplitude volume) in stratigraphic interpretation, especially in case of reservoirs having less thickness and/or gradational impedance contrast.

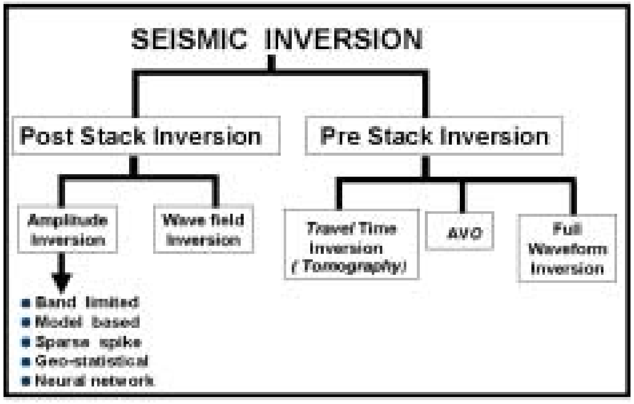

Fig.1: Different seismic inversion techniques.

Presently, acoustic impedance obtained from post stack stratigraphic inversion is being extensively used in hydrocarbon exploration and exploitation activities. Post stack seismic inversion techniques work mostly on the basis of one dimensional convolution model of seismic trace, which forms the basis for inversion. Post stack seismic inversion combines geophysical, geological and petrophysical data through a robust inversion scheme to extract the more reliable stratigraphic information and reservoir properties from the seismic data (Van Riel, 2000, Pramanik et al., 2001, Vig et al., 2002). In practice, stratigraphic inversion is being carried out through two approaches (1) linear inversion using layer stripping scheme and (2) non-linear iterative least square technique. Non-linear techniques are being widely used for inversion because linear inversion is highly unstable and noise sensitive. Non-linear least square inversion minimises squared residual error between the observed and computed seismic data for the layered earth model considering plane source. The starting model gets updated iteratively with the incremental update of the model parameters. Such approach is logically appealing, but it faces difficulties due to inadequate robustness that restrict guess model to be in the close neighbourhood of true model. Elastic impedance is the latest advancement for converting reflection amplitude into elastic impedance using global optimization techniques such as simulated annealing, genetic algorithms, taboo search etc., from pre-stack seismic data (Sen and Stoffa, 1995; Connoly, 1999; Mallick, 1999; McCormack *et al*., 1999; Verwest, 2000). These techniques do not require any initial guess model and have significantly improved the robustness of inversion but require extensive computational efforts.

In this paper, various benefits of using acoustic impedance over conventional seismic data have been outlined. In order to provide pre-inversion checks, possible measures to ensure the validity of assumptions and the compatibility of the data with velocity/ impedance constraints, theoretical background, a complete methodology incorporating all the steps of practical post stack stratigraphic inversion and different inversion schemes used in the industry have been described in detail. Further, to demonstrate the application of this complete inversion sequence and to provide the real insight to the working geoscientists, some of the synthetic and real data examples of post stack stratigraphic inversion have been illustrated.

Benefits of stratigraphic inversion:

Stratigraphic inversion provides acoustic impedance from seismic data. Acoustic impedance (AI) is defined as the product of rock density and P-wave velocity in one dimensional earth model. As a result, AI makes sequence stratigraphic analysis more straight forward because:

1. Acoustic impedance is a rock property and represents layer properties where as seismic amplitude is an interface property and represents a contrast between layers (relative measurement). It is less oscillatory than the initial seismic data.

2. Acoustic impedance is an absolute measurement using priori information where as true seismic amplitudes are not always preserved and often-seismic data is not zero phased.

3. Acoustic impedance can be directly measured at well locations where as seismic amplitude cannot be measured directly.

4. Acoustic impedance is mostly free from noise and thin layer interference where as seismic amplitude is affected by the presence of noise and thin layer interference.

5. Acoustic impedance is free of wavelet effects and is obtained after integration of data from several sources like seismic, well logs and geology. Wavelet side lobes are attenuated, eliminating some false stratigraphic-like effects in acoustic impedance where as they are present in case of conventional seismic data and mislead towards erroneous interpretation.

6. Acoustic impedance has been considered as an ideal parameter for volume based geologic interpretation. This allows fast and accurate delineation of target bodies and can be directly related to well control, allowing the use of porosity, saturation and lithology logs to analyse their variation away from the well locations in 3-D area.

7. Acoustic impedance produces a high-resolution layer framework, where the strata are deformed in order to become conformable to seismic reflectors. Understanding of earth subsurface in terms of layers and layer properties is more straightforward for geologists and reservoir engineers than in terms of seismic reflectors.

8. Acoustic impedance improves the visualisation through layering and enhances vertical resolution where as conventional seismic data does not.

9. Acoustic impedance concept can be generalised to carry out the inversion of angle or offset stack data to elastic impedance or elastic parameters. Elastic impedance provides AVO information and, in conjunction with AI, improves interpretation power and ability to discriminate lithology and fluids.

Theory related to stratigraphic inversion:

The inversion of post stack seismic data to obtain time domain pseudo acoustic impedance logs is not new. Its basic procedure was reported for the first time by Lavergne (1975) and subsequently by Lindseth (1979). In order to recapitulate and provide basic understanding of seismic inversion, it has been briefly described here. In a laterally homogeneous acoustic media, seismic wave propagation can be approximately modeled by one dimensional convolution theory. In one dimensional convolution model, a seismic trace is result of the convolution of earth's reflectivity with a seismic source function along with additive noise component. In equation form, a seismic trace can be written as

$$S_t = W_t * R_t + n_t \qquad (1)$$

where S_t = the seismic trace
W_t = seismic wavelet
R_t = earth reflectivity
n_t = additive noise component

Basically, each reflectivity or reflection coefficient may be thought of as the response of the seismic wavelet to an acoustic impedance change within the earth, where acoustic impedance (I_P) is defined as the product of compressional velocity (V_p) and density (r). For normal incidence, the reflection coefficient ($R_{P,\,i}$) at the ith interface is given as

$$R_{P,\,i} = (I_{P,\,i+1} - I_{P,\,i}) / (I_{P,\,i+1} + I_{P,i}) \qquad (2)$$

where $I_{P,\,i+1}$ and $I_{p,\,i}$ are the acoustic impedances in the (i+1)th and ith layers respectively. The acoustic impedance $I_{P,\,i+1}$ can be obtained from reflectivity equation (2) by recursive formula as

$$I_{P,\,i+1} = I_{P,\,i}\,(1 + R_{P,\,i}) / (1 - R_{P,\,i}) \qquad (3)$$

From this equation, it is clear that if full band reflectivity ($R_{P,\,i}$) and acoustic impedance ($I_{P,\,i}$) of a reference layer is known , one can easily recover the acoustic impedance trace from the seismic reflection amplitude. Due to computational efficiency, this one dimensional convolution model of seismic trace has widely been used in most of the post stack seismic inversion methods (Lines and Treitel, 1984, Oldenburg et al, 1986). A schematic diagram showing seismic inversion from a noise free seismic trace based on one dimensional convolution model is shown in Figure(2). But there are several assumptions in the one dimensional convolution model, which require attention before using it for inversion. These assumptions are:

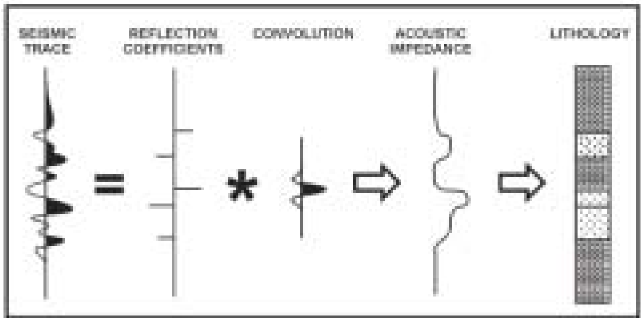

Fig. 2 : Schematic diagram showing post-stack seismic inversion.

1. Each seismic trace represents true zero offset reflectivity directly below the seismic trace locations.

2. Seismic amplitudes are truly recovered and preserved during processing.

3. There are no multiples and all the seismic amplitudes of trace correspond to primaries only.

4. There are no AVO effects. If the seismic data does contain AVO anomaly, the inversion process will attribute the entire effect to the changes in density and velocity rather than Poisson's ratio.

5. Seismic data is free of coherent noise. The noise component present in the data is purely random and uncorrelated with seismic.

6. Seismic wavelet is constant (preferably zero phase) and does not change with time.

Methodology for stratigraphic inversion :

Different methodologies have been proposed by the industry to perform such stratigraphic inversion from post stack seismic data. Here an attempt has been made to provide a brief outline for carrying out post stack stratigraphic inversion and its flowchart is given in Figure(3). Various steps mentioned in flowchart for post stack inversion are described below separately:

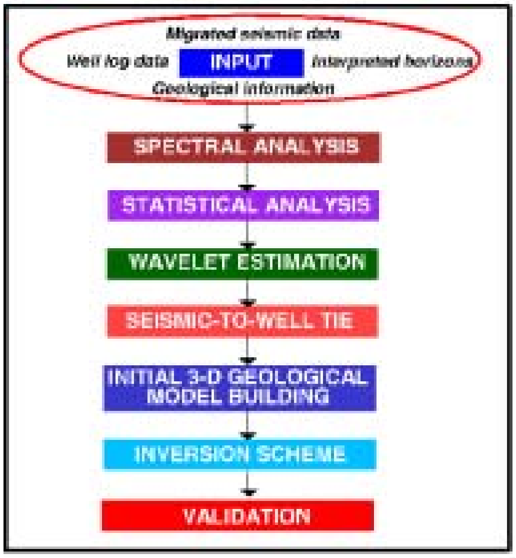

Fig. 3 : Flow Chart of post-stack seismic inversion.

A. Spectral analysis: Generally, It is assumed that in the finally processed data, seismic amplitude is flat and zero phase in usable frequency bandwidth where as in reality it is not. Therefore, prior to seismic inversion, it is very important to carryout the spectral analysis of seismic traces at different locations in 3D data volume to ascertain the reliable spectral bandwidth, signal to noise ratio and phase variation. This analysis of amplitude, phase variation and signal to noise ratio plays a crucial role during wavelet estimation, in reduction of uncorrelated noise through filtering techniques for enhancing S/N ratio and during phase correction of processed seismic data into zero phase. The zero phase seismic wavelet contains maximum energy in its central lobe only.

B. Statistical analysis: Prior to the stratigraphic inversion, a statistical analysis of well log data is necessary to perform for understanding the relation between seismic parameters, acoustic impedance, velocity and density. This analysis uses petrophysics, log data and cross-plots to determine the relationships and provide a solid base for prediction of lithology in distinguishing reservoir and non-reservoir shaly facies. Further, it also helps in establishing the relation between the reservoir property and each of the acoustic properties that are affected by changes in the reservoir property.