Fast Vertical Positioning with GPS & EGM96 Geoid Model

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Summary

This paper aims in introducing a modern technique for determining elevation, avoiding cumbersome and time consuming spirit leveling operations.

Fast vertical positioning can be obtained using GPS with Geoid model. Relative accuracy of GPS derived orthometric with EGM96 (Geoid model currently available for the entire world) will be of the order of 6-9 ppm approx. in mountainous areas. This accuracy is good enough for oil exploration projects such as seismic survey and drilling rig positioning.

If the bench marks with known elevations are less than 5 to 10 km apart, and if the area is not too hilly, then the geoid may in some cases be approximated by a plane, and no special geoid information may be required. In this case, GPS alone can be used to obtain precise height differences with precision of the order of 1 to 3 cm for less than 10 km.

Introduction

Seismic surveying demands elevations of upholes and pkts well in advance for planning, generating models (NSM), calculating static corrections and pre-processing of data in FPU etc. Introduction of mobile processing unit for online seismic quality monitoring also demands elevation data in advance.

At present elevation along seismic profiles and upholes is being determined by the conventional method of spirit leveling. In this, differential leveling will be carried out from the known elevation station (B.M) to the required stations, whose elevation has to be determined and closed at the same or another B.M for checking. This involves more time and cost.

A new method of leveling with GPS & geoid model can be used, since GPS can provide more accurate horizontal and vertical relative positioning. Elevations can also be obtained instantaneously by using the state-of-art technology DGPS with geoid model. This will fulfill the requirements of providing elevations in advance for seismic surveying.

The method and technique involved in obtaining elevation with GPS & geoid model and achievable accuracy will be discussed in detail. A case study in this regard was carried out in the highly undulating and hostile terrain of Mizoram and the results discussed.

Leveling e with GPS

In many oil exploration applications the conventional method of spirit leveling is being replaced with GPS and geoid model. To understand the technique of GPS leveling, first lets begin with conventional leveling.

Geodetic leveling (conventional spirit leveling)

In this method of leveling a leveling instrument containing an aiming telescope is aligned at a right angle to local gravity by means of a bubble. One looks through the telescope, which defines the local level plane, and reads two graduated level rods, obtaining a level difference between them. The level difference depicted in figure 1 depends not only on the topography, but also on local variation in the direction of gravity.

The upper part of the figure 1 shows leveling on planar topography in a planar gravity field. The lower part of the figure portrays different level reading one obtains due to gravity variation induced by a mass concentration. This shows that the leveled heights not only reflects terrain variations, but also gravity variations.

Orthometric heights (H) are referenced to geoid (equipotential surface at mean sea level) The orthometric height of a point on the Earth’s surface is the distance from the geoid to the point, measured along the plumb line normal to the geoid. Ellipsoid heights (h) are referred to a reference ellipsoid. The ellipsoid height of a point is the distance from
the reference ellipsoid to the point, measured along the line which is normal to the ellipsoid. At a given point, the difference between its ellipsoid height and orthometric height is defined as its geoid height (or) geoidal undulation (N).

According to figure 2.

\[ H = h - N \]  ———— (1)

and, using differential GPS positioning between two points,

\[ \Delta H = \Delta h - \Delta N \]  ———— (2)

The GPS provides only \( h \) or \( \Delta h \). To convert these mathematical ellipsoidal quantities to physically meaningful orthometric ones, the geoid undulations \( N \) or \( \Delta N \) must be computed from gravity data or from an accurate geoid model. If this is not done, error exceeding several meters may result.

To achieve \( \Delta H \) accuracies which are comparable to \( \Delta h \), the \( \Delta N \) component must of the same quality. Typical accuracies of the GPS component \( \Delta h \) are of the order of 1 to 2 ppm. Various studies have shown that \( \Delta N \) can also be obtained with the same accuracies. Consequently, \( \Delta H \) accuracies of 1 to 2 ppm are also possible, rendering leveling by GPS with accurate geoid model a very good (much faster and cheaper) alternative to the traditional spirit leveling.

As mentioned earlier geoid undulation is nothing but the geoid – ellipsoid separation, obtained using gravimetric observations. If the local gravity data is not available then a geopotential model geoid solution may be used but high accuracies cannot be expected.

Geoidal undulation \( N \) is calculated by a geopotential model \( GM \), mean free air gravity anomalies \( \Delta g \) and heights \( H \) in a digital terrain model DTM is based on the following formula.

\[ N = NGM + N\Delta g + NH \] ———— (3)

In rough topography all the three data are necessary for estimating \( N \). The following diagram shows the contribution of different data to \( N \).

From figure 3 it is clear that the gravity anomalies \( N\Delta g \) have the contribution of the topography. Mostly strokes integral equation is used to calculate \( N\Delta g \). Methods to calculate all the three components of geoid undulation \( N \) is discussed in standard books.

The procedure of GPS leveling in practice would be to obtain \( \Delta h \) from the GPS solution, compute \( \Delta N \) gravimetrically and then derive \( \Delta H \) using equation (2).
When there is at least one station in the area with known elevation (orthometric height), say $H_0$, the orthometric elevation of all other stations can be derived using $H_0$ and the computed $\Delta H_s$.

**Actual field condition and feasible solution**

To calculate all the three component of geoid undulation $N$, one should have

- Precise spherical harmonic coefficients of the geopotential model (NGM)
- Gravity measurements at an average spacing of 3 km or less ($N\Delta g$)
- DTM's of closer spacing. (NH)
- Fast fourier transformation FFT software to calculate gravimetric data.

Geopotential models like JGM-3, OSU91A, EGM96 are being used for NGM. For our discussion and case study only EGM96 is used since it gives better results over the other two old models.

The following table shows the estimated $\Delta N$ accuracies of a Canadian model. Almost same results can be expected from EGM96 model.

<table>
<thead>
<tr>
<th>Area Type</th>
<th>GM</th>
<th>GM + $\Delta g$</th>
<th>GM + $\Delta g + H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat</td>
<td>5-7</td>
<td>1-3</td>
<td>1-3</td>
</tr>
<tr>
<td>Mountain</td>
<td>6-9</td>
<td>3-5</td>
<td>1-3</td>
</tr>
</tbody>
</table>

From table (1) it is understood that, if no local gravity data is available, then using geopotential model like EGM96 alone will give favorable accuracy of approx. 6-9 ppm in mountainous area. (i.e. 3- 4 cm error for a baseline of 4 km), which may be good enough for topographical survey.

**Earth Gravitational Model 1996 (EGM96)**

EGM96 is a geopotential model of the Earth consisting of spherical harmonic coefficients complete to degree and order 360. It is a composite solution, consisting of: (1) a combination solution to degree and order 70; (2) a block diagonal solution from degree 71 to 359; and (3) the quadrature solution at degree 360. This model is the result of a collaboration between the National Imagery and Mapping Agency (NIMA), the NASA Goddard Space Flight Center, and the Ohio State University.

The joint project took advantage of new surface gravity data from many different regions of the globe, including Africa, Canada, parts of South America, Greenland and parts of the Arctic and the Antarctic, Southeast Asia, Eastern Europe, and the former Soviet Union. In addition, there have been major efforts to improve NIMA's existing 30' mean anomaly database through contributions over various countries in Asia.

Other data that contributed to EGM96 are altimeter-derived anomalies from ERS-1 and from the GEOSAT Geodetic Mission (GM), extensive satellite tracking data - including new data from Satellite laser ranging (SLR), the Global Positioning System (GPS), NASA's Tracking and Data Relay Satellite System (TDRSS), the French DORIS system, and the US Navy TRANET Doppler tracking system - as well as direct altimeter ranges from TOPEX/POSEIDON (T/P), ERS-1, and GEOSAT.

EGM96 model is used to compute geoid undulations accurate to better than one meter (with the exception of areas void of dense and accurate surface gravity data) with reference to WGS84 ellipsoid.

Additional results from the EGM96 solution include models of the dynamic ocean topography from T/P and ERS-1 together, and GEOSAT separately, and improved orbit determination for Earth-orbiting satellites. Figure 4. shows the EGM96 geoid undulations with reference to WGS84 ellipsoid, which varies from – 105.0 m to 85.0 m

**GPS / Leveling Comparisons**

The following results shows the difference in elevation (absolute) obtained using GPS with different geoid models and geodetic leveling, from many parts of the world.

![Fig. 4: EGM96 15 minute Geoid](image-url)
Comparisons of Geoid undulations from geopotential models using 298 stations in British Columbia.

<table>
<thead>
<tr>
<th>Geopotential Model</th>
<th>Mean Difference (m)</th>
<th>Standard Deviation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JGM-3/OSU91A</td>
<td>-1.060</td>
<td>0.575</td>
</tr>
<tr>
<td>EGM96</td>
<td>-1.118</td>
<td>0.521</td>
</tr>
</tbody>
</table>

(NGS dataset, USA)

Comparisons of Geoid undulations from geopotential models using the Milbert stations (April 1996) in the U.S. [Thinned 1156 stations], from Rapp [1996].

<table>
<thead>
<tr>
<th>Geopotential Model</th>
<th>Mean Difference (m)</th>
<th>Standard Deviation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JGM-3/OSU91A</td>
<td>-0.236</td>
<td>0.947</td>
</tr>
<tr>
<td>EGM96</td>
<td>-0.219</td>
<td>0.517</td>
</tr>
</tbody>
</table>

(China)

The total number of the local GPS/Leveling points is 238

<table>
<thead>
<tr>
<th>Geopotential Model</th>
<th>Mean Difference (m)</th>
<th>Standard Deviation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSU91A</td>
<td>0.687</td>
<td>1.510</td>
</tr>
<tr>
<td>JGM3OSU</td>
<td>0.782</td>
<td>1.389</td>
</tr>
<tr>
<td>EGM96</td>
<td>0.004</td>
<td>0.689</td>
</tr>
</tbody>
</table>

Mizoram, India (A case study)

To check the performance of EGM96 and the feasibility of using GPS for fast vertical positioning, a case study was carried out in a highly undulating and hostile terrain of Mizoram. Leica SR520 12 channel dual frequency GPS receivers were used for all static observations. Initially a GPS network was carried out, covering the entire working area of GP-10, connecting SOI GTS station at Bualpui Kawnpui and SOI BM at Kawnpui.

GPS data processed using SKI-Pro processing software and network adjusted by least square technique. Transformation parameters for converting coordinates from WGS 84 to Everest 1830 ellipsoid was calculated by Bursa-Wolf 7 parameter method. EGM96 geoid model was then attached to the GPS project for obtaining geoidal undulation and GPS-derived orthometric heights.

Figure 5 shows the GPS network with geoid undulation contour of 15 cm interval. This geoid undulation is nothing but the first component (NGM) of equation (3) from the geopotential model EGM96.

The second component (NΔg) of geoidal undulation (N) of equation (3) can be calculated from the local gravity data (Δg) using Fast fourier transformation software (FFT). Due to the non availability of surface gravity data and software, our case study was limited to using only the first component of equation (3) from EGM96.

Fig. 5: GPS Network in Mizoram area of GP-10
For comparing GPS/leveling, GPS connection was made to SOI BM at Kawnpui, and Departmental Bench marks (DBM) at GPS117, GPS 117A and TBM-1 (figure 6), having a GPS base at Govt.PS Kawnpui. Also 11 pickets were connected with GPS, whose elevations were made available with conventional method of spirit leveling using Auto level.

Discussion of results

Comparison of GPS /SOI BM from the above table shows that the mean difference in elevation between GPS and SOI BM values is 2.766 M in absolute mode and 3.2 cm for a distance of 425 m in relative mode. Long baseline cannot be checked because of non availability of BMs in long ranges.

Few pickets of known elevation with Auto leveling were also connected with GPS and compared, the mean difference in elevation was 3.266 in absolute mode and 0.458 m for a baseline of 7.988 km in differential mode. The error is slightly on the higher side, may be because of rough terrains, methodology and instruments used for conventional spirit leveling differing from SOI higher order leveling. Hence these statistics from Auto level data cannot be treated as reference, only been used for academic interest.

<table>
<thead>
<tr>
<th>Bench Mark / DBMs</th>
<th>Elevation by leveling (M)</th>
<th>Elevation by GPS (M)</th>
<th>Diff. (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BM KAWNPUI (SOI)</td>
<td>846.670</td>
<td>849.409</td>
<td>2.739</td>
</tr>
<tr>
<td>GPS117</td>
<td>844.410</td>
<td>847.183</td>
<td>2.773</td>
</tr>
<tr>
<td>GPS117A</td>
<td>847.369</td>
<td>850.155</td>
<td>2.786</td>
</tr>
<tr>
<td>TBM-1</td>
<td>826.104</td>
<td>828.873</td>
<td>2.769</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>2.766</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pickets</th>
<th>Elevation by Auto level (M)</th>
<th>Elevation by GPS (M)</th>
<th>Diff. (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L01-PKT 115</td>
<td>44.529</td>
<td>47.826</td>
<td>3.297</td>
</tr>
<tr>
<td>PKT432 L-01</td>
<td>218.699</td>
<td>221.875</td>
<td>3.176</td>
</tr>
<tr>
<td>PKT99 L-02</td>
<td>135.209</td>
<td>138.466</td>
<td>3.257</td>
</tr>
<tr>
<td>PKT-422 L-02</td>
<td>104.109</td>
<td>107.746</td>
<td>3.637</td>
</tr>
<tr>
<td>PKT-570 L-02</td>
<td>136.994</td>
<td>140.123</td>
<td>3.129</td>
</tr>
<tr>
<td>P-3 L-03</td>
<td>158.540</td>
<td>161.974</td>
<td>3.434</td>
</tr>
<tr>
<td>L03TBM</td>
<td>206.419</td>
<td>208.618</td>
<td>2.199</td>
</tr>
<tr>
<td>PKT-162 L-04</td>
<td>51.619</td>
<td>55.577</td>
<td>3.958</td>
</tr>
<tr>
<td>PKT-307-L-04</td>
<td>127.034</td>
<td>130.547</td>
<td>3.513</td>
</tr>
<tr>
<td>PKT-434 L-05</td>
<td>137.169</td>
<td>140.099</td>
<td>2.930</td>
</tr>
<tr>
<td>HORTOKI-1</td>
<td>73.749</td>
<td>77.144</td>
<td>3.395</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>3.266</td>
</tr>
</tbody>
</table>
Table 4. shows the expected results of elevation using GPS and EGM96 geoid model in Mizoram area.

<table>
<thead>
<tr>
<th>Geopotential Method</th>
<th>Mean Difference (m)</th>
<th>Standard Deviation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGM96 Absolute</td>
<td>2.766</td>
<td>0.020</td>
</tr>
<tr>
<td>EGM96 (expected)</td>
<td>9-10 ppm i.e. 3-4 cm (for 4 km baseline)</td>
<td></td>
</tr>
</tbody>
</table>

**Conclusions**

From various GPS/Leveling test results of British Columbia, USA, China and our case study in Mizoram, India it is clear that GPS with EGM96 gives ± 1 to 3 M absolute vertical positioning accuracy and 6-9 ppm accuracy (expected) in differential mode. So if there is at least one BM of known elevation available in the area, then elevation of other stations can be obtained with 3 to 4 cm accuracy for a baseline of 4 km in a differential mode. It is always advisable to connect several bench marks, if available and check for misclosures. However the baseline should be restricted to 4 to 5 km for better accuracy, since troposphere modeling and filtering is a must for longer baselines for achieving good vertical accuracy.

The study was taken by to realize the concept of deriving orthometric heights from GPS. The results obtained were quite encouraging, which confirms the hypothesis that the technique has the potential to replace the cumbersome process of spirit leveling. Thereby ensuring value addition in seismic surveying and oil exploration in particular.

However a local gravimetric model for Indian region is the utmost necessity to perform more authentic and accurate conversion of GPS ellipsoidal heights to spirit leveled heights.

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Views expressed in this paper are that of the author(s) only and may not necessarily be of ONGC.

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