2D DC Resistivity Data Interpretation Using Analytical Signal Approach

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Summary

Our 2-D stabilized analytic signal algorithm (RESAS) is used in interpreting synthetic electrical profiling data due to 2-D resistive/conductive bodies of both closed and open geometries. For bodies of closed geometry (Vertical prism and two inclined prisms), secondary pole-pole profiles with buried current pole above body center are considered while for open geometries (Step fault, Horst, Double step fault and horst – graben structures), pole-pole (Half - Wenner) / pole-dipole apparent resistivity profiles are opted for. The analytic signal parameters like, Amplitude of the Analytic Signal (AAS), Real (RIAS) and imaginary parts (IIAS) of complex analytic signal inverse are used in the analysis. Synthetic buried pole-pole data is generated by finite-difference based resistivity modeling algorithm and this data served as input to RES2AS. Achieved results show that AAS generally meets the interpretation requirements; but analyses of RIAS and IIAS are needed either to confirm the lateral coordinates inferred by AAS or supplement such information in case AAS fails. Analytical signal method fails for body corners with same lateral coordinates (bodies with vertical dips). Further, numerical results confirm that profiles of sufficient length are needed to image the lower corners of anomaly causative sources. The obtained results validate the proposed interpretation procedure. The method is also tried successfully on a mining case study.

Introduction

Resistivity imaging is currently utilized for tackling a wide variety of near surface geophysical problems. The state-of-art resistivity imaging algorithms either deals with improved pseudo-section concepts (Loke & Barker 1995) or computer-intensive resistivity inversions (Li & Oldenburg, 1994). However, there is a practical need for simple and effective means of implementing resistivity inversion of profile data under field conditions. The current effort is devoted to this aspect.

Dey and Morrison (1979) used the theoretical aspects of Finite-difference (FDM) based forward modeling by Mufti (1976, 1978) and provided an algorithm (RESIS2D), which involves a 3D current source over a 2D earth model. In the present effort this algorithm (Dey, 1976) is used for pole-pole and pole-dipole configurations.

Analytic Signal method (Nabighian, 1972), which has shown success in magnetic data interpretation, can also be applied for dc resistivity data (Pujari, 1998; Pujari and Sastry, 2003; Sastry and Pujari, 2005).

The complex analytic signal computation depends on stable numerical derivatives of secondary electric potentials. So, Thikhonov’s regularization is incorporated in the designed analytic algorithm. Theoretical modeling is carried out on isolated 2D conductive/resistive bodies of rectangular cross-section enclosed within resistive/conductive host medium. Here, physical property (electrical resistivity) is not being assessed.

Theory

Stabilized Analytic Signal Method

The construction of analytic Signal involves numerical derivative evaluations from the input potential field data. However, such numerical derivative computations are unstable (Tikhonov and Arsenin, 1977). Hence, one needs to employ a regularization strategy for evaluation of stable numerical derivatives, which in turn will lead to a stable Analytic Signal computation.

2-D Analytic Signal

The analytic signal is defined as a complex function whose real and imaginary parts constitute a Hilbert Transform pair.

Following Nabighian (1972), the 2-D analytical signal, \( A(X, Z) \) of the secondary pole-pole potential \( V_s(x, z) \) or apparent resistivity can be defined as
\[ A(x, z) = \frac{\partial V'}{\partial x} + j \frac{\partial V'}{\partial z} \]  
(1)

where \( j = \sqrt{-1} \)

The amplitude of the analytic signal is given by

\[ |A(x, z)| = \sqrt{\left( \frac{\partial V^{'2}}{\partial x} \right)^2 + \left( \frac{\partial V^{'2}}{\partial z} \right)^2} \]  
(2)

where \( V'_x = \frac{\partial V'_s}{\partial x} \)

The real and imaginary parts of R.H.S. of eqn. (1) constitute a Hilbert Transform pair (Nabighian, 1972).

Our spectral algorithm (RESAS), an updated version of Pujari and Sastry (2003) uses both regularization strategy (Tikhonov and Arsenin, 1977) and FFT routines. In RESAS a more effective FFT routine from Press et al. (1994) is used and the rest of the logic remains unchanged.

**Parameters of Analytic Signal**

Nabighian (1972) has identified the following properties of complex Analytic Signal of magnetic anomaly due to a 2-d body of arbitrary cross-section, approximated by an n-sided polygon.

- Amplitude of Analytic Signal (AAS).
- Amplitude of the Real part of \( 1/A(x, z) \) i.e., (RIAS).

The AAS is used for defining body corners (both depth and lateral coordinates). However, for confirmation of lateral coordinates or when AAS fails RIAS and IIAS, revealing the zeros of the inverse of analytic signal (AS), are used. Here, we use both AAS and RIAS plots for interpretation.

**Methodology**

It involves input data preparation for our RES2AS and analysis of resulting analytic signal parameters. The charge accumulation concepts of Li & Oldenburg (1991) stress the need of secondary potential estimation in resistivity profile interpretations for conductive / resistive targets of closed geometry.

In case of isolated closed bodies located within a layered medium, secondary potential, \( V'(x) \) is calculated by subtracting the layered half space potential, \( V^l(x) \) from observed total potential, \( V^t(x) \) i.e.,

\[ V^t(x) = V^l(x) - V^l(x) \]  
(3)

We found from synthetic models that there is no need of secondary potential generation due to bodies of open geometry and one can directly work with apparent resistivity data itself.

**Depth Rules**

Unlike magnetics (Nabighian, 1972) no depth rule exists for electrical resistivity case. We have found that the depth rule, \( d = x^{1/2} \) for AAS over a single corner is alright for bodies of closed geometry. For bodies of open geometry, empirically depth rules are designed on the basis of numerical experiments. Further, the depth \( d=2x^{1/2} \) can be adopted for bodies of open geometry.

**Numerical Experiments and Results**

**Closed Bodies**

Initially, we consider the geological models of close geometry whose synthetic apparent resistivity data is generated by RESIS2D for various electrode configurations and current source locations. Secondary potential is calculated and interpreted through RES2AS.

**Single Vertical Conductive Prism In Layered Medium**

Single conductive prism in layered medium is shown in Fig. 1. Secondary potential is calculated using RES2D for vertical prism (Fig.1). Point current source is located at (36, -1) above the body center of Prism. Depth of current source is located 1m below air-earth interface Fig. 2 Secondary potential plot for source above the body centre of prism.

**Interpretation**

The synthetic secondary potential (Fig.2) served as input to RES2AS and the resulting AAS (Fig.3) and RIAS (Fig. 4) are interpreted for body parameters, shown in Table1. Figure5 summarizes the interpretation results.

**Open Bodies**

Bodies of open geometry are also treated by analytic signal approach and methodology is found to be of reasonable success.
Step Fault

Step fault model (Fig. 6) is considered as open body. The forward response (Fig. 7), the pole-pole apparent resistivity profile is determined for an electrode spacing of 5m. with the configuration run at a depth of 3m.

Interpretation

AAS (Fig. 8), RIAS (Fig. 9) and IIAS (Fig. 10) are analyzed as per earlier outlined theory to determine depth and location of fault.

Case Study

Figure 12 shows apparent resistivity contour data obtained from a survey in eastern Nova Scotia (Telford et al. 1976) for base metal exploration. The electrode arrangement was double-dipole.

Local Geology

The rocks in the area are generally volcanic although there are no outcrops; the overburden is not
Table 1: Results of Stabilized Analytic Signal (AS) Algorithm for Vertical Prism Model (Fig. 1)

<table>
<thead>
<tr>
<th>Body Corner</th>
<th>RIAS X-Coordinate (m.)</th>
<th>AAS Actual Body Coordinate (X, Z), (m.)</th>
<th>Coordinate (m.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>28</td>
<td>(27,4)</td>
<td>(28,4)</td>
</tr>
<tr>
<td>Second</td>
<td>44</td>
<td>(43,4)</td>
<td>(44,4)</td>
</tr>
</tbody>
</table>

Fig. 5: Inferred Model (broken line) by AAS

Fig. 7: Apparent resistivity profile for step fault model with current source at depth 1 m. (Fig. 6)

Fig. 8: Amplitude of Analytic Signal (AAS) for Step Fault model (Fig. 6)

Table 2: Summary of fault model results

<table>
<thead>
<tr>
<th>Body Corner</th>
<th>RIAS X-Coordinate (m.)</th>
<th>IIAAS X-Coordinate (m.)</th>
<th>AAS Actual Body Coordinate (X-Z), (m.)</th>
<th>Actual Body Coordinate (m.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>41</td>
<td>41</td>
<td>(45,7)</td>
<td>(40,9)</td>
</tr>
</tbody>
</table>
expected to be anywhere more than 25 ft deep and usually is less than 15 ft. There is a large-scale geochemical anomaly (Cu, Pb, Zn) associated with the area. Drainage is to the south while the glaciation direction is approximately northeast. The problem entails in characterizing polymetallic sulphide body (or bodies).

By considering the behavior of low resistivity contour 200 &-f, profiles 40N, 44N, 48N and 56N are selected for analysis.
Input Data

Four profiles 40N, 44N, 48N and 56N in Fig 12 formed input to RES2AS. The resulting AAS and RIAS plots are analyzed and the results are shown in Table 3.

Interpretation

The lateral width are revealed by RIAS in Table 1 has outlined the body rich in polymetallic sulphides as shown in Fig. 13. Depths to top surface of body along profiles 40N, 44N, 48N and 56N (Table 7.1) are utilized in construction of a 3-D image of inferred conductive body (Fig. 14).

Table 3: Summary of Case Study Results

<table>
<thead>
<tr>
<th>Interpretation using RESAS (Fig 7.1)</th>
<th>RIAS (X-Coordinate) (m)</th>
<th>AAS ((X, Z), Coordinate) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile 1</td>
<td>300450</td>
<td>(300,25)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(460,27.5)</td>
</tr>
<tr>
<td>Profile 2</td>
<td>365400525</td>
<td>(360,35)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(400,35)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(520,35)</td>
</tr>
<tr>
<td>Profile 3</td>
<td>250335</td>
<td>(258,22.5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(340,32.5)</td>
</tr>
<tr>
<td>Profile 4</td>
<td>180362</td>
<td>(180,30)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(360,70)</td>
</tr>
</tbody>
</table>

Conclusions

Tikhonov’s regularization based stable analytic signal algorithm has provided first order guess models for input error-free pole-pole potential data in case of conductive bodies of closed geometry and resistivity data for conductive structures of open geometry. While secondary pole-pole potential data of closed 2-D bodies formed the input data to our RES2AS for 2D bodies of open geometry pole-pole resistivity profiles suffice. Generally, AAS meets the interpretation requirements; but analysis of RIAS and IIAS is needed either to confirm the lateral coordinates inferred by AAS or supplement such information.

In case of body corners with same lateral coordinates (bodies with vertical dips), analytical signal method fails in segregating them and ambiguity arises. Profiles of sufficient length are needed to image the lower corners of anomaly causative sources. Numerical experiments demonstrate the use of the proposed method.

Acknowledgements

Authors thank Dr. V.N.Singh, Professor & Head, Department of Earth Sciences, IIT, Roorkee for providing necessary facilities in carrying out the reported work.
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