Ambiguities in Seismic Wave Velocity Analysis and in its AVO Response in Gas Hydrate Bearing Sediments

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ABSTRACT: Seismic investigation in different continental margins successfully imaged the bottom simulating reflectors (BSRs), which signify the base of the methane hydrate stability field. Though in many cases the presence of free gas beneath the BSR have been reported, the free gas may or may not be present depending upon the geological situation. The predicted seismic velocities above the BSR in hydrate bearing sediments are very high compared to that of the free gas-bearing zone. Different empirical relationships have been proposed to fit the estimated velocity from the seismic data. But none of the existing techniques can unambiguously describe the nature of variation of P and S-wave velocities with hydrate saturation as well as free gas saturation because of complex nature (origin and occurrence) of gas hydrate sediments. A weighted equation based on the three-phase time average and Wood equations has been applied by Lee et. al. (1996) to derive a relationship between the compressional wave (P wave) velocity and the amount of hydrates filling the pore space. Behaviour of the weighted equation in the porosity range of 40-80% is satisfactory but the different constants involved in the method are confusing in their physical significances. To explain the high seismic velocity in hydrate bearing sediments, Dvorkin and Nur (1993, 1996) have proposed contact cement model concept. The effective medium theory is being attempted to explain this issue also. Based on these velocity models, AVO responses using original Zoepritz equations are generated to estimate the hydrate saturation above BSR as well as the existence of free gas beneath it. The relative AVO responses of these velocity models are presented here. Also the merits and demerits are discussed from the rock physics concept.

INTRODUCTION

The objective of this paper is to compare theoretically the amplitude variation with offset for different velocity models from the point of rock physics. The simple question, which arises, is “why do we need a proper velocity model in gas hydrate bearing sediment?” The answer is very straightforward. We know that the BSR is the demarcation between hydrated sediment above and free gas zone beneath. But there is no prominent seismic signature at the top of the hydrate bearing sediments. So it is very difficult to quantify the amount of hydrates present in the oceanic sediments. Again the core drilling cannot help in quantification procedure for the time being, because hydrate is highly vulnerable to destabilization in laboratory condition. Besides, there are speculations of probable escape of methane gas in the atmosphere when hydrates are destabilised. Under these constraints we need to simulate an alternative method to predict the hydrate saturation from a velocity model. The AVO method can be applied successfully to estimate hydrate saturation.

THEORY

1) Partition of energy at the interface

The effect of boundaries on wave propagation is complex. Not only reflected and refracted waves are generated at the boundaries but also these waves are interconvertible from one form to another depending upon the angle of incidence. At larger incidence angle this effect is very clear. As a result of this, the reflection coefficients vary. The Zoepritz equation determines the amplitudes of the reflected and refracted waves. The Fig. 1 illustrates the mode conversion of the incident seismic wave. The Zoepritz equations can be put in matrix form (Waters, 1977) which is more suitable for computer solution : $Q=P^{-1}R$ where P is 4 x 4 square matrix whose elements are comprised of P and S wave velocities of the media, incident angle, reflection (incident as well as converted waves) and refracted angles and densities of the media.

AVO responses from Zoepritz equation

The fundamental link between seismic velocities with rock properties for a homogeneous, isotropic elastic medium are given by the following formula :

$$V_p = [(K + (4/3)\,G) / \rho] \quad \text{(i)}$$

$$\text{And } Vs = (G / \rho)^{1/2} \quad \text{(ii)}$$

Where,

Vp = Compressional wave (P-wave) velocity
Vs = Shear wave (S-wave) velocity
K = Bulk modulus
G = Rock shear modulus which governs the rock rigidity
ρ = Rock bulk density

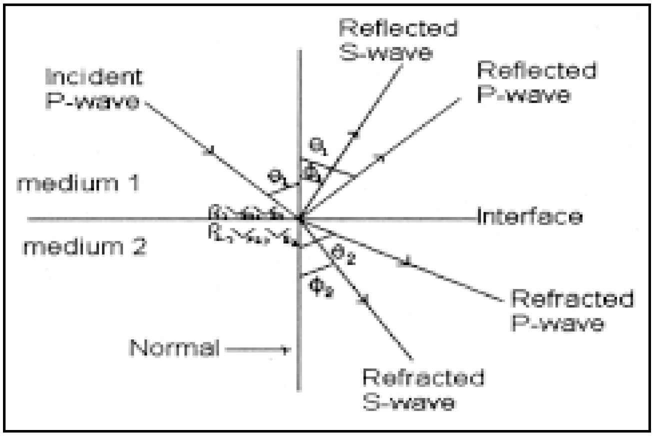

Figure 1: Angular relations between incident, reflected, and refracted rays when a wave encounters a boundary between two different solid media.

It has been demonstrated many times (e.g., Domenico, 1976, 1977) that the rock bulk modulus may be strongly dependent on the pore fluid bulk modulus. The above fundamental equations are not directly applicable to determine the seismic velocities in hydrate as well as gas bearing sediments. This is because the hydrate and gas inclusion in the formation change overall elastic behaviours of the fluid-filled porous solid reservoir (Geertsma, 1961). For a gas bearing formation of this kind, seismic velocities at infinite frequency (Minshull et al., 1994) are given by :

$Vp = \{Vp_1 \times Vp_2\}^{1/2}$ and $Vs = G_b / V$

Where,

Vp = P- wave velocity, and Vs = S-wave velocity,

$Vp_1 = \{[1/C_b + (4/3)\, G_b + (\phi\rho_b / k\rho_f + (1-\beta\,(1-\beta-2\phi/k)) / (1-\phi- \beta)\, C_s+ \phi C_f)],\ Vp_2 = 1/ (\rho_b\, (1-\rho_f\phi/\rho_b K)),$

$V = \rho_b\, (1-\phi\, \rho_f /K\rho_b),\ C_b = (1-\phi)\, C_s + \phi C_p,\ C_f = (1-S_g)\, C_w + S_g C_g,\ C_p = C_{p0} + M/P_d,\ \rho_b = \phi\rho_f + (1-\phi)\rho_s,\ \rho_f = S_w\rho_w + (1-S_w)\rho_g,\ \rho_d = (\rho_{av} - \rho_w)\, gZ,\ \beta = C_s/C_b$

C_b is compressibility of the reservoir matrix (frame) devoid of fluid,

C_f is compressibility of the pore fluid,

ρ_b is the reservoir bulk density,

ρ_s is sediment grain density (2.65×10^3 kg/m^3),

ρ_f is density of pore fluid,

ρ_w is sea-water density (1.03×10^3),

ϕ is fractional porosity,

K is a factor, between one and infinity, describing the degree of coupling between pore fluid and frame,

Cs is compressibility of solid grains ($2.50 \times 10_{-11}$ Pa^{-1}),

C_w is compressibility of water (4.19×10^{-10} Pa^{-1})

C_g is compressibility of gas, here methane (CH_4) gas (4.24×10^{-8} Pa^{-1}),

C_{p0} is pore compressibility at zero differential pressure (2.96×10^{-9} pa^{-1}),

M is pressure gradient of pore compressibility (-7.5×10^{-17} Pa^{-2}),

G_b is shear modulus of the reservoir frame (1.19×10^9 Pa),

Z is depth of BSR below sea bed (600m)

ρ_w is density of methane at certain depth,

ρ_{av} is mean density of rock above BSR (1.80×10^3 kg/m^3),

K is coupling factor (2.5),

g is acceleration due to gravity (9.81 m/sec^2),

C_p is pore compressibility,

S_g is gas saturation = $(1- S^w)$, S_w is watre saturation,

The seismic velocity above the BSR in hydrate bearing zone is given by different empirical relationships.

A) Three-phase weighted equation for the compressional wave velocity is given by Lee et al., 1996 :

$$\frac{1}{V_p} = \frac{W\phi\,(1 - S)^n}{V_{p1}} + \frac{1 - W\phi\,(1 - S)^n}{V_{p2}} \qquad (1)$$

Where, Vp = compressional velocity of hydrate bearing sediments: V_{p1} = compressional velocity of hydrate-bearing sediment computed from the three-phase Wood equation; V_{p2} = compressional velocity of hydrate-bearing sediment computed from the three phase time-average equation; W:a weighting factor; ϕ : sediment porosity (as a fraction); S: concentration of hydrate in the pore space (as a fraction); n: a constant simulating the rate of lithification with hydrate concentration. If the composite system consists of four phases (e.g., clay, sand, hydrate and pore fluid) then the above weighted equation is to be replaced by four-phase weighted equation considering four-phase Wood equation and four-phase time average equation.

The shear wave (S-wave) velocity in hydrate bearing sediments (Lee et al., 1996) is given by :

$$V_s = V_p\,[\alpha\,(1-\phi) + \beta\phi S + \gamma\phi\,(1-S)] \qquad (2)$$

With $\alpha = V_s / V_p|_{matrix}$, $\gamma = V_s / V_p|_{fluid}$

There are different proposed empirical relationships (Castagna et al, 1985) for calculating shear-wave velocity from P-wave velocity. These are

V_s (Km/s) = 0.8042 V_p - 0.8559 for sandstone (2a)

V_s (Km/s) = 0.7700 V_p - 0.8674 for shale (2b)

Variation of seismic velocities (P and S wave) and Poisson's ratio with saturation (both hydrate and gas) computed using the above equation are shown in Fig. 2.

B) Effective medium equation

The hydrate-saturated rock may be regarded as composite system of different constituents. The mineralogical contributions in conjunction with hydrate are visualized as an effective medium to give rise the same response in velocity as it would have been due to the system. Hydrate may be considered as a part of the fluid or as a part of the dry fame depending upon the hydrate saturation (Helgerud et al., 1999). If it is a part of fluid then its effect in S-wave velocity becomes insignificant. On the other hand, when it is assumed as a part of dry frame then increase of S-wave velocity can be explained. Under this circumstance, gas hydrate becomes load-bearing components of the frame (matrix) and the porosity is reduced by the gas hydrate saturation. The reduced porosity is defined as $\phi_f = (1-S)\phi$, S being the hydrate concentration. Here the concept of critical porosity also comes into picture.

At critical porosity the effective bulk modulus (K_{hm}) and shear modulus (G_{hm}) of the dry rock frame are computed using the Hertz-Mindlin contact theory (Mindlin, 1949)

$$K_{hm} = \left[m2\,(1-\phi_c^{\,2})\, G^2P / 18\pi^2\,(1-v)^2 \right]^{1/3}$$

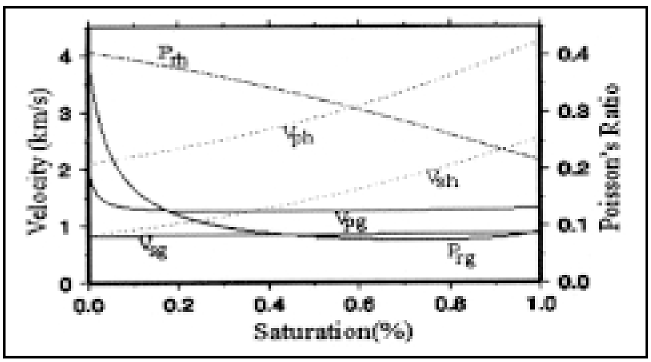

Figure 2: Seismic velocities (V_p and V_s) and Poisson's ratio (P_r) in ocean bottom sediments at typical BSR depths (porosity-0.4) using the equations of Biot (1956) and Geertsma (1961) as quoted by Domenico (1977). Subscripts 'h' and 'g' symbolize for hydrate and gas bearing sediments respectively.

$$G_{hm} = \left[\frac{(5\ -\ 4\,v)}{5\ (2-v)} \right] + \left[\frac{3\ m^2\,(1-\phi_c^{\,2})\,G^2P}{2\,\pi^2\ (1-v)^2} \right]^{1/3} \qquad (3)$$

Where P is the effective pressure; K, G, and v are the matrix bulk modulus, shear modulus and the Poisson's ratio respectively; m is the average number of contacts per grain in the sphere packing of random orientation.

$P = (\rho_b - \rho_w)\, gD$, where ρ_b is the bulk modulus of sediment, ρ_w is water density, g is gravitational constant and D is the depth.

The bulk modulus (K_{dry}) and shear modulus (G_{dry}) of the rock frame when the porosity (ϕ) of rock frame is below critical porosity ($\phi<\phi_c$) are given by the modified lower bound of Hashing-Shtrikman (Dvorking and Nur) :

$$K_{dry} = \left[\frac{\phi/_{\phi_c}}{K_{hm} + \frac{4}{3}G_{hm}} + \frac{1-\phi/_{\phi_c}}{K + \frac{4}{3}G_{hm}} - \right] \frac{4}{3}\,G_{hm}$$

$$G_{dry} = \left[\frac{\phi/_{\phi_c}}{G_{hm} + Z} + \frac{1-\phi/_{\phi_c}}{G + Z} \right]^{-1} - Z$$

$$\text{Where } Z = \frac{G_{hm}}{6} \left[\frac{9\,K_{hm}\ +\ 8\,G_{hm}}{K_{hm}\ +\ 2\,G_{hm}} \right] \qquad (4)$$

For $\phi>\phi_c$

$$K_{dry} = \left[\frac{(1-\phi)}{(1-\phi_c)} \left[\frac{1}{\left[K_{hm} + \frac{4}{3}G_{hm}\right]} \right] + \frac{(\phi-\phi_c)}{(1-\phi_c)} \left[\frac{1}{\frac{4}{3}G_{hm}} \right] \right]^{-1} - \frac{4}{3}\,G_{hm}$$

$$G_{dry} = \left[\frac{(1-\phi)}{(1-\phi_c)} \left[\frac{1}{G_{hm} + Z} \right] + \frac{(\phi - \phi_c)}{(1-\phi_c)\,Z} \right]^{-1} - Z \qquad (5)$$

In these expressions ϕ is reduced porosity ϕ_f

For sediment saturated with pore fluid of bulk modulus K_f, the shear modulus G_{sat} remains the same but bulk modulus (K_{sat}) changes. This is calcuted from Gassman's (1951) equation as

$$K_{sat} = K \left[\frac{\phi K_{dry} - (1+\phi)\,K_f\,K_{dry}\,/K + K_f}{(1-\phi)\,K_f + \phi K - K_f\,K_{dry}/K} \right] \text{ And } G_{sat} = G_{dry} \qquad (6)$$

For a matrix consisting of different minerals K and G of equation (3) are calculated via Hill's (1952) average formula:

$$K = (1/2)\left[\sum_{i=1}^{L} f_i K_i + \left(\sum_{i=1}^{L} f_i / K_i\right)^{-1}\right]$$

$$G = (1/2)\left[\sum_{i=1}^{L} f_i G_i + \left(\sum_{i=1}^{L} f_i / G_i\right)^{-1}\right] \quad (7)$$

Where L is the number of mineral constituents and fi is the volumetric fraction of the i-th constituent having K_i and G_i in the solid phase. The solid phase density (ρb) are computed as

$$\rho = \sum f_i \rho_i \quad \text{and} \quad \rho_b = (1-\phi)\rho + \phi\rho_w \quad (8)$$

Where, ρ_i is the density of i-th constituent. Because the hydrate is a part of the matrix, the volumetric fraction of constituents fi in the above cases must be replaced by fi ⇒ fi $(1-\phi)/(1-\phi_f)$ and the hydrate volumetric fraction is given by :

$$f_h = S\phi/(1-\phi_f)$$

Equation (6) is used for calculating P and S-wave velocities when the pore space is filled with water via the following equation :

$$VP = \left[\frac{K_{sat} + \frac{4}{3}G_{sat}}{\rho_b}\right]^{1/2} \quad \text{And} \quad V_S = \left[\frac{G_{sat}}{\rho_b}\right]^{1/2}$$

Variation of seismic velocities (P and S wave) and Poisson's ratio with hydrate saturation computed from effective medium theory and have been compared with that of the weighted equation are shown in Fig. 3.

DISCUSSION

AVO responses using the original Zoepritz equation using the above two methods have been shown in Figure 4. The lithologies (e.g., densities, mineralogical compositions) have been kept the same so that the amplitude variation can be compared using the two techniques. Also the values of weighting factors (W and n) used in four-phase weighted equation have been adjusted to get nearly the same seismic velocities and densities for the sediments above and beneath the BSR, when there is no hydrate or free gas. This is required because in absence of both hydrate above the BSR and free gas beneath the BSR there shouldn't have any impedance contrast to give rise to reflection. To obtain the AVO response, the four-phase weighted equation has been considered

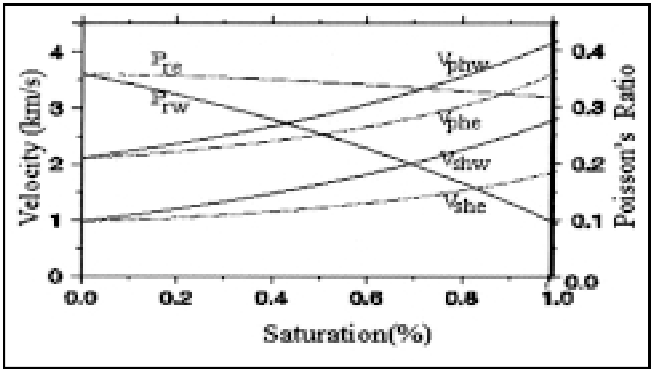

Figure 3: Comparison of seismic velocities from four-phase weighted equation and effective medium theory (dashed curves). The subscripts 'w' and 'e' symbolize for hydrate and gas bearing sediments respectively. Subscripts 'h' and 'g' symbolize for hydrate and gas bearing sediments.

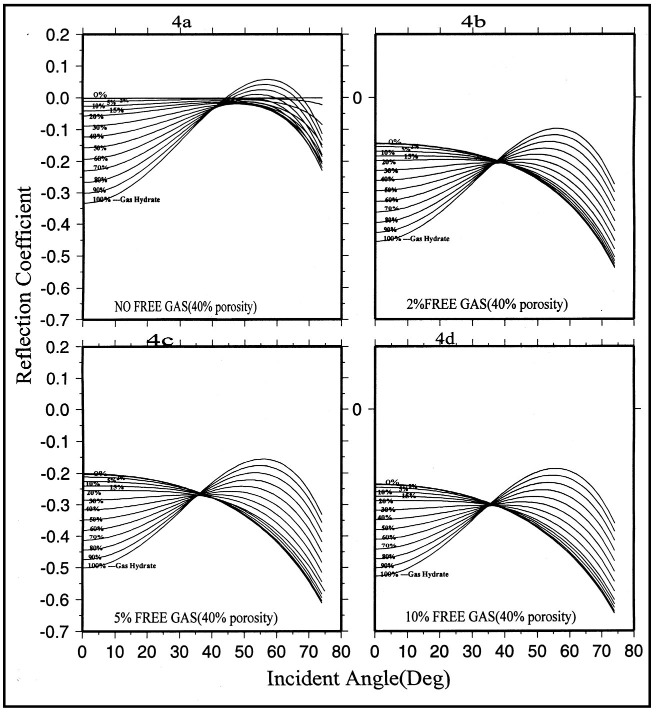

Figure 4: The AVO responses computed from the four-phase time average equation.

instead of three-phase equation. The matrix constituents are sand (70%) and clay (30%). The elastic constants and other parameters used here are given in the Table 1. From the Fig. 4a and 4b it is clear that a very small inclusion of free gas (even 2% or less) in the sediments beneath the BSR causes a drastic change in the AVO responses. But, further increase of free gas (even 10% or more) beneath the BSR doesn't give rise to

Table 1 : Elastic constants and some relevant parameters used for velocity models.

Name	Symbol Used	Values Used	Sources
Weight	*W*	1.44	*Lee and Collett* (1999)
Exponent	*n*	1	*Lee and Collett* (1999)
V/V_p for non hydrate-bearing sediment	α	0.56	*Lee and Collett* (1999)
V/V_p for hydrate	β	0.51	*Sloan* (1998)
Volumetric Clay Content	Cv	0.3	From well logs
Shear Modulus of Quartz	G_s *(Gpa)*	45	*Heigerud et al.* (1999)
Bulk Modulus of Quartz	K_s*(Gpa)*	*36*	*Heigerud et al.* (1999)
Shear Modulus of Clay	G_t*(Gpa)*	6.85	*Heigerud et al.* (1999)
Bulk Modulus of Clay	K_g*(Gpa)*	*6.85*	*Heigerud et al.* (1999)
Shear Modulus of Hydrate	G_h*(Gpa)*	2.54	**Sloan* (1998)
Bulk Modulus of Hydrate	K_h*(Gpa)*	*6.41*	**Sloan* (1998)
Density of Quartz	ρ_s(g/cm^3)	2.65	*Heigerud et al.* (1999)
Density of Clay	ρ_s(g/cm^3)	2.58	*Heigerud et al.* (1999)
Density of Hydrate	ρ_h(g/cm^3)	0.91	*Sloan* (1998)
Critical Porosity	ϕ_c	0.38	*Nur et al.* (1998)
Number of contacts per grain	*m*	9	*Dvorkin and Nur* (1996)

*The shear and bulk moduli are computed from the P-wave and S-wave velocities with the density given in *Sloan* [(1998)] for structure I type gas hydrate.

any significant change in AVO trend, excluding the increase in absolute normal incidence reflection coefficient. Again from the above figures, it is seen that above 30% (around) hydrate saturation, existence of free gas beneath the BSR doesn't change the AVO trends. So it can be possible to estimate hydrate saturation from the shape of the AVO curve, and only the existence of the free gas beneath the BSR can be determined if the hydrate saturation above the BSR is less than 30%. The AVO patterns from the effective medium theory (Fig. 5) are not showing any clear change due to presence of free gas bellow the BSR. Therefore, it seems that four-phase equation approach (which accounts of fundamental elastic behaviour of the composite formation of different lithology) is more suitable than of the effective medium theory.

CONCLUSION

The AVO analysis in hydrate bearing sediment differs from the conventional AVO analysis in hydrocarbon bearing sediments in the sense that in later case the change in velocity is due to lithology change in the successive formations whereas in hydrate bearing sediment there is no change in

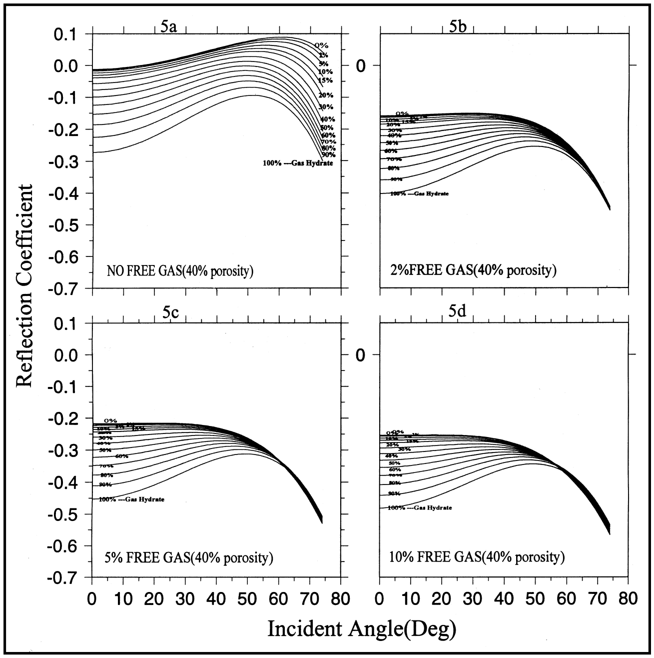

Figure 5: The AVO responses computed from the effective medium theory

lithology. In order to demarcate the hydrate bearing sediment above and gas bearing sediment below from the AVO analysis we need to have an authentic seismic velocity structure. Also, we need some models/proper equations that can solve the problem in determining seismic velocities in ambiguous hydrated sediments as well as the estimation of free gas saturation beneath the BSR, even when the hydrate saturation above the BSR is more than 30%. Travel time topography and 2-D waveform inversion in association with rock physics concept may be invoked to derive the seismic velocity function in hydrated sediment very accurately.

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