Joint Inversion of Seismic First Arrival Travel Time and Gravity Data – An Application of Arc Tangent Basis Function and Very Fast Simulated Annealing

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ABSTRACT: Joint interpretation of disparate geophysical datasets is essential to deriving meaningful subsurface geologic models. Here we describe a method for joint inversion of first arrival travel time and gravity data with application to field data from a geologically complex subduction zone. We employ a layer-based model description, in which interfaces (which may also be called isovelocity lines) are defined by summation of arc-tangent functions. Within each layer, the velocity is assumed to vary linearly with depth at each surface location. Density is related to velocity using a fourth order polynomial whose coefficients are assumed known. The non-linear optimization problem is solved by a very fast simulated annealing (VFSA). We applied our technique to field data collected over the Ryukyu subduction zone offshore Taiwan during an ocean bottom seismometer (OBS) experiment (called TAICRUST) conducted in the year 1995. Application to one NS trending line resulted in acceptable fit of both travel time and gravity data. The resulting model clearly delineates the plate boundaries.

INTRODUCTION

There exist large classes of geological models, which can cause anomalies in several geophysical fields. This is the reason that simultaneous interpretation of multi-set geophysical data to obtain an integrated model, is becoming more popular. Such an integrated model is definitely a more geologically feasible solution than what is obtained by individual analysis. Moreover, combined use of different methods helps to fill up the gap of one method by the other. Thus, to capitalize on the advantages of different methods at a time and to constrain the geological model, information from different surveys have been combined in the past to provide a more complete picture of the subsurface geology. Although integrated interpretation of geophysical and geological data is routinely done using ‘trial and error’ forward modeling, formal simultaneous inversion of disparate data sets is not used widely. Menichetti and Guillen (1983) and Zeyen and Pous (1993) used joint inversion of gravity and magnetic data to reduce the ambiguity in interpretation. Zhao (1995) performed weighted joint inversion of GPS and gravity data to delineate the active fault segment movement at Red River fault zone. Salichon et al (2003) used both seismic and InSAR data jointly to limit the trade-off between the space and time aspects of the rupture velocity of Peru earthquake. The most popular and widely explored field of cooperative interpretation is combined use of both gravity and seismic data. Seismic and gravity methods can compliment each other in various ways.

Gravity is a powerful method for delineation of shallow structures as its amplitudes (or its kernel function) decay rapidly with depth. On the other hand, broad band seismic survey is not that sensitive to shallow structures, but effective for deeper structures. Further, gravity method is sensitive to lateral variation of mass distribution only; whereas, sharp vertical variation in structures can be detected by well-designed seismic experiments only. Lines et al (1988) discussed various ways of cooperative inversion by describing the differences between joint inversion and sequential inversion. Strykowski (1999) precisely pointed out some technical details about the mathematical formulation of joint seismic-gravity inversion problem. Oppenheimer and Herkenhoff (1981) modeled velocity structure from inversion of teleseismic data and used linear velocity-density relation for three dimensional gravity modeling of Geysers-Clear lake region of California. Joint inversion of teleseismic delay time and gravity anomaly is used by Savino et al (1980) and Kuhn et al (2002) for better interpretation of the data. Lees and VanDecar (1991) also used linear relation between density and velocity (Brich’s law) to perform simultaneous inversion of both gravity and travel time data. Nielsen and Jacobsen (2000) presented an integrated inversion scheme for crustal modeling by using wide angle seismic and gravity data.

In the present work, we simultaneously use the wide-angle seismic travel time and the gravity anomaly to delineate the crustal structure of the subduction zone of Taiwan.
the area is very complex and needs sufficient constraints to restrict the model within possible geology. During TAICRUST experiment (1995) organized by 14 different institutions and universities of Taiwan, USA and France, MCS and OBS (4 component) data were collected simultaneously along six lines. Here, we show the interpretation along line 1, which is parallel to the Ryukyu trench-arc-backarc system – the most active convergent margin in the circum-Pacific belt. We performed weighted joint inversion by using very fast simulated annealing technique (VFSA) and obtained a satisfactory agreement between travel time, as well as, gravity data.

**METHOD**

The main objective of our work is to combine information from seismic first arrival travel time and gravity data to obtain subsurface images that explain both the observations. Note that the travel time and gravity data are sensitive to different physical parameters, which are generally related and can be considered as outcomes of two different experiments. Let \( T \) and \( g \) represent vectors of travel time and gravity anomalies respectively. Let \( \alpha(x) \) and \( \rho(x) \) represent spatial distributions of compressional wave velocity and density respectively and \( x \) represent a position vector. We have,

\[
T = f_1 \{ \alpha(x) \} \quad (1)
\]

and

\[
g = f_2 \{ \rho(x) \} \quad (2)
\]

In equations (1) and (2), \( f_1 \) and \( f_2 \) describe the forward modeling operators for travel time and gravity respectively, described later in the text. We now denote \( T_{\text{obs}} \) and \( T_{\text{syn}} \) to represent observed and synthetic travel time data respectively. Similarly we denote \( g_{\text{obs}} \) and \( g_{\text{syn}} \) to represent observed and synthetic gravity data respectively. Thus we seek the distributions \( \alpha(x) \) and \( \rho(x) \) such that the functional

\[
F(\alpha(x), \rho(x)) = (T_{\text{obs}} - T_{\text{syn}})^T C_T^{-1} (T_{\text{obs}} - T_{\text{syn}}) + \omega (g_{\text{obs}} - g_{\text{syn}})^T C_g^{-1} (g_{\text{obs}} - g_{\text{syn}}) \quad (4)
\]

attains a minimum. The superscript \( T \) denotes a matrix transpose and the matrices \( C_T \) and \( C_g \) are the data covariance matrices for travel time and gravity respectively.

**Model Parameterization**

Based on the results obtained from tomographic inversions in such areas, we define our model space such that they consist of a few distinct layers. We allow for tremendous flexibility in the definition of our interfaces, which are essentially iso-velocity lines.

Following Sen and Frazer (1995), we define an interface in 2D using a sum of arc-tangent functions in horizontal distance \( x \) (Figure 1), such that

\[
z(x) = z_0 + \sum_{k=1}^{n} \Delta Z_k \left[ 0.5 + \frac{1}{\pi} \tan^{-1}\left( \frac{x-x_k}{b_k} \right) \right] \quad (5)
\]

where \( z \) is the depth, \( n \) is the number of arc-tangent nodes, \( z_0 \) is an average depth of the interface, \( x_k \) is the horizontal location of an arc-tangent node, and \( \Delta Z_k \) is the vertical throw attained asymptotically over a horizontal distance of \( b_k \).

The entire model space is defined by a set of interfaces. In addition to searching for the arc-tangent parameters, we also search for velocities above and below the interface. The velocity and density of the water layer are assumed to be 1.5 km/sec and 1.0 respectively. Across the interfaces at each horizontal location, the velocity is interpolated at each grid point by linear interpolation. This results in velocity values at each grid location in the model.

These velocities are converted to densities using the following formula

\[
\rho(x,z) = a_1 + a_2 \alpha(x,z) + a_3 \alpha^2(x,z) + a_4 \alpha^3(x,z) + a_5 \alpha^4(x,z) \quad (6)
\]
where the coefficients are given by 

\[ a_1 = 0.6997, \quad a_2 = 2.23, \quad a_3 = -0.598, \quad a_4 = 0.0703, \quad a_5 = -0.00283 \]

**FORWARD MODELING**

**Travel time computation**

Here we have employed the scheme proposed by Schneider, Jr. et al (1992) for computing first arrival travel times for crustal velocity models for use in the VFSA inversion. Schneider, Jr. et al (1992) use a simple calculus based technique to compute traveltime and makes no assumption on velocity smoothness. The traveltime computation begins with the starting values computed near a source location. Then mapping systematically steps through the grid, where each new traveltime is calculated using two previously computed neighbor traveltimes. After eight calculations at each grid point, the minimum time is assigned to the grid. At any stage during the mapping, only the most recently computed traveltime is used to calculate new traveltimes. Schneider Jr. et al (1992) propose two mapping procedures, a brute force approach that advances across the grid one column (or row) at a time and a more natural approach that computes times along expanding rectangles.

**Computation of Gravity Anomalies**

In this application we are interested in computing gravity anomalies from a grid of rectangular blocks, each with a constant density, distributed uniformly in the subsurface. For computing the gravity response of each rectangular block we employed the method of Talwani (1959). The gravity field at the ith observation point can be computed from the co-ordinates of each corner of the polygon and can be expressed as

\[ g_i = \sum_{j=1}^{K} \left( F(x_j, z_{i1}) + F(x_j, z_{i2}) \right) \tag{7} \]

where \( K \) is the total number of corners of the polygon and \( F(x_j, z_{i1}) \) and \( F(x_j, z_{i2}) \) are the nonlinear functions (Talwani et al, 1959) associated with the co-ordinates of the four points defining the rectangular block.

We use very fast simulated annealing (Kirkpatrick et al, 1983; Sen and Stoffa, 1995) for inversion of the above defined objective function. Inversion procedure involves the following steps:

- Define interfaces using arc-tangent nodes,
- Choose reasonable search limits for the node parameters, mean depth and the velocities,
- Given a set of parameters, obtain velocity values at every grid location in \((x,z)\),
- Compute density values at every grid location \((x,z)\) from these velocity values using Eq. (6),
- Compute theoretical travel times using Eikonal equation,
- Compute gravity anomalies using Talwani’s method,
- Evaluate the objective function using Eq. (4),
- Compute a model update using VFSA rule
- Go to step 3 and continue till convergence.

**FIELD EXAMPLE**

We will now demonstrate an application of our algorithm to field dataset consisting of travel time observations recorded on several ocean bottom seismometers and gravity data collected in the area of the subduction zone of Taiwan during TAICRUST experiment in 1995. The understanding of the Taiwan Orogen is still limited due to the absence of information about the geometry of the crust near plate boundary. The crsital structure of Taiwan has been studied by several scientists (e.g., Shih et al, 1998; Chen, 1995). We found some strong disagreements between the results obtained by different workers. Thus, with the goal of deriving a model of crustal structure that is consistent with seismic and gravity observations, we employed our inversion algorithm to data set from Line 1.

**Inversion of seismic data alone**

We first applied our inversion algorithm to first arrival travel times picked from seven ocean bottom seismometers.
placed along the line as shown in figure 2. The model space was defined with 6 interfaces each with 7 arc-tangent nodes. Figure 3 shows the travel time data fit obtained with our best-fit VFSA model.

Next we converted our best-fit VFSA model to a corresponding density model using the density-velocity relation as stated above and the gravity anomaly computed for this model is compared against gravity observations in Figure 4. We notice that there are large mismatches between the two.

**Joint inversion of seismic and gravity data**

Next, we apply the present technique of joint inversion to OBS first travel time and gravity data. Figure 5 compares the observed and computed travel times, whereas, the match between the computed and observed gravity data is displayed in Figure 6. Note that although the fit between the observed and synthetic travel time is slightly degraded compared to the results obtained from travel time inversion alone, there is substantial improvement in the match between observed and computed gravity anomalies.

**CONCLUSION**

The inherent ambiguity in geophysical interpretation can always lead us to a completely unrealistic model by showing an excellent agreement between observed and computed data. Combined use of several different geophysical datasets can help us in constraining the model. In the present work we used seismic travel time data and gravity data jointly to obtain a well constraint subsurface geology. The arc tangent function is found to be useful to model the smooth as well as nearly discontinuous changes in depth. The mismatch between
Joint Inversion of Seismic First Arrival Travel Time

observed gravity anomaly and the anomaly computed from the density model obtained from travel time inversion only indicates the underlying ambiguity present in travel time inversion. Joint inversion of travel time and gravity data is able to find out a common model having an optimum fit of both travel time and gravity data. Such constrained model is definitely more realistic than any other models obtained from individual inversion.

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