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Fractional Derivative in Pressure Transient data Analysis with Phase Redistribution in Fractal Reservoir for Reservoir Characterization

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Summary

The present paper describes the fractional derivative use in a pressure transient data analysis with phase redistribution for fractal reservoir in a geological environment which is not possible by conventional methods. The application of pressure derivatives in pressure transient test data interpretation by reservoir engineers is common place since the work done by Bourdet et al. in 1989 on this subject. The analysis of such type of data in reservoir characterization is well known as inverse problem. The fractal geometry in a dynamic pressure transient tests data plays a vital role for heterogeneity characterization.

Understanding fractures is important for exploration and production in naturally fractured reservoirs. The existence of where fractures are and how the flowing ones are interconnected through fracture network modeling is beneficial for effective planning of hydrocarbon recovery measures. The goal of the pressure transient data interpretation is to establish a reasonable estimate of the reservoir properties of interest for better understanding of reservoir behavior. Recent experiences have brought forward more reasonable expectations of geologists, geophysicists, petrophysicists and reservoir engineers by sharing their knowledge in integrating various sources of measured data in drilled wells in finding the reservoir heterogeneity. The pressure transient test and production data are with reservoir engineers and it may provide further clues about the nature of the fracture network within a reservoir.

The computer aided matching technique for both pressure and its derivative are used in estimating the reservoir properties from measured pressure data of heterogeneous reservoir. By a process of iteration the geological and flow data can be reconciled to produce plausible models which could help to constrain reservoir simulation.

The acquisition of pressure transient reservoir data and their continued evaluation at different locations with geo-statistical methods are foundational to sound the reservoir management to develop the field and implement the applicable improved oil/enhanced oil recovery schemes.

Introduction:

In reservoir with a low permeability matrix the majority of the fluid flow is along fractures. The stress acting on these fractures can have a significant effect on the flow of fluid through fracture network. It has been shown by many researchers that the transmissivity of the fracture varies with normal stress acting on the fracture. For fluid flow through a fracture network it is necessary to consider both static connectivity of the fracture network based on length, orientation and spatial distribution of the fractures and the dynamic connectivity of the network the influence of stress

acting on fluid flow through the fracture network (as given in Figures A and B).

Fractional calculus is a branch of mathematics which deals with application of derivatives and integrals of real order. It is an old subject since its development started from Leibniz and Euler. In recent years, the fractional derivatives have been used to model physical processes, leading to the formulation of fractional differential equations. Fractional calculus has been very popular by its wide applications with fractal which includes viscoelasticity, electro analytical chemistry, biology, physics, pressure transient in



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oil industry & etc. The use of fractional derivative for modeling pressure transient data analysis in fractal reservoir is quite natural. A growing number of increasingly sophisticated measurements have demonstrated that some variations in reservoir deliverability are related to interactions between changing fluid pressure, reservoir stresses, and natural fracture permeability during production or injection. The sensitivity to changing stresses is probably most pronounced in tight, over pressured, naturally fractured reservoirs where the elasticity of rock and large pressure changes can cause significant changes in fracture apertures. Fractures in such reservoirs may both dilate during injection and close during drawdown. The mechanical properties of the strata in some reservoirs are such that entirely new fracture sets can be created solely by production related stress changes.

The various researchers have shown that the transport in fractally fractured media using the anomalous diffusion theory in the sense that the complex geometry slows down the particle motion (Chang and Yortsos [1], Acuna and Yortsos [2]). Park et al [3] have improved it by proposing the modified equation to allow the variation in flow path and the memory of the fluid flow.

Examination of various processes in fractal medium usually results into the equations in fractional derivative, which is defined by Oldham and Spanier [4]. It is generally accepted that for spatially correlated media such as fractally fractured reservoirs, the memory of the fluid and non-locality are very important in all stage of production. It is known that fractional calculus successfully describes such characteristics of the anomalous diffusion. There are two major approaches of fractional calculus for the study of anomalous diffusion. The latter is known to fully describe diffusional feature relatively well. The fractional calculus was introduced for the analysis of fractal behavior in transport process. Fractional derivatives make it possible to consider the memory of the property by definition (Podlubny [5]).

Mathematical Formulation for the flow equation

We begin with the definition of a fractional derivative and its application since they are used in the formulation of the problems. Several definitions of a fractional derivative and integral have been proposed. The most frequently used definition of a fractional derivative of order $\alpha > 0$ is the

Riemann-Liouville definition, which is straightforward generalization to non-integer values of Cauchy formula. The Riemann-Liouville fractional derivative is defined as [5]

$$D^{\alpha} f(t) = \begin{cases} \frac{d^n}{dt^n} \left[\frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \right], n-1 < \alpha < n \\ \frac{d^n}{dt^n} f(t), \alpha = n \end{cases} \quad (1)$$

Where, Γ is the Gamma function, n is positive integer such that $n-1 < \alpha < n$ and $\alpha > 0$.

An alternative definition of the fractional derivative was proposed by Caputo [5]

$$D^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{d^n f(\tau)}{d\tau^n} d\tau, n-1 < \alpha < n \\ \frac{d^n}{dt^n} f(t), \alpha = n \end{cases} \quad (2)$$

The major utility of this type of fractional derivative is caused by the treatment of differential equations of fractional order for physical applications, where the initial conditions are usually expressed in terms of a given function and its derivatives of integer order, even if the governing equation is of fractional order. Similarly, introducing the arbitrary positive real number α the fractional integral of order $\alpha > 0$, defined as

$$J^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad t > 0, \quad \alpha \in \mathbb{R}^+ \quad (3)$$



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It is important to emphasize that what seems to be really interesting in studying fractals via fractional calculus, are the non-integer physical dimensions that arise dealing with both fractional operators and fractal sets. The mathematical formulation of pressure transient equation for fractal reservoir where the fracture network is largely divided by backbone fractures and fractal fracture loops has been given by Metzler et al [6]. The mass balance equation over fractal loop has been derived by Park et al. [3] and the radial diffusion equation for pressure in dimensional forms is given below

$$\frac{\partial^\gamma P_{fD}(r_D, t_D)}{\partial t_D^\gamma} = \frac{1}{r_D^{d-1}} \frac{\partial}{\partial r_D} \left[r_D^{d-\theta-1} \frac{\partial P_{fD}(r_D, t_D)}{\partial r_D} \right] \quad (4)$$

Where, $0 \leq \gamma < 1$ from the definition of fractional diffusion equation of order, γ . This equation reduces to standard diffusion equation, when $\gamma = 1$, $\theta = 0$, and $d = 1, 2, 3$ for Euclidean dimension.

The initial condition is given as follows

$$P_{fD}(r_D, 0) = 0 \quad (5)$$

The inner boundary conditions are given below

$$P_{wD} = [P_{fD} - S r_D^{d-\theta-1} (\partial P_{fD} / \partial r_D)]_{r_D=1} \quad (6)$$

$$C_D (dP_{wD} / dt_D) - [r_D^{d-\theta-1} (\partial P_{fD} / \partial r_D)]_{r_D=1} = 1 \quad (7)$$

The outer boundary condition is that of an infinite acting reservoir and is given as

$$P_{fD}(r_D, t_D) \rightarrow 0, \text{ as } r_D \rightarrow \infty \quad (8)$$

The Fair [7] has modified the van Everdingen and Hurst [8] equation of sand face rate for constant wellbore storage effect by adding a term to account the pressure change due to phase redistribution with the result being

$$(q_{sf}/q) = q_{wD} = 1 - C_D [(dP_{wD}/dt_D) - (dP_{\phi D}/dt_D) - C_{mD} \frac{d^2 q_{wD}}{dt_D^2}] \quad (9)$$

Thus, phase redistribution was modeled as a changing wellbore storage phenomenon. The pressure function $P_{\phi D}$, has the following properties

$$\lim_{t \rightarrow 0} P_{\phi D} = 0 \quad (9a)$$

$$\lim_{t \rightarrow \infty} P_{\phi D} = C_{\phi D}, \text{ a constant} \quad (9b)$$

$$\lim_{t \rightarrow \infty} (dP_{\phi D} / dt_D) = 0 \quad (9c)$$

Application of increasing/decreasing wellbore storage model to field data was first used by Fair as exponential form for changing storage pressure function as

$$P_{\phi D} = C_{\phi D} (1 - e^{-t_D / \alpha_D}) \quad (10a)$$

and Hegeman et al. [9] has assumed the changing storage pressure as



$$P_{\phi D} = C_{\phi D} \operatorname{erf}(t_D / \alpha_D) \quad (10b)$$

Where, $C_{\phi D}$ and α_D are changing storage pressure parameter and changing storage time parameter. The other users have found the class of function as in equation (10b) that obeys properties (9a) - (9c), was found to exhibit characteristics representative of the field data.

Solution of the Problem

The partial differential equation (4) and (10) together with initial and boundary conditions (5) - (8) are solved with the use of Laplace transformation. The Laplace transform of fractional derivative of order $D^\alpha f(t)$ is given as

$$L[D^\alpha f(t)] = s^\alpha F(s) - \sum_{j=0}^{n-1} s^j [D^{\alpha-j-1} f(t)]_{t=0} \quad (11)$$

Where, $F(s)$ is the Laplace transform of, $f(t)$.

Alternative the Laplace transform for the Caputo fractional derivative is as

$$L[D^\alpha f(t)] = s^\alpha F(s) - \sum_{j=0}^{n-1} D^j f(0^+) s^{\alpha-1-j}, n-1 < \alpha < n \quad (11a)$$

The Caputo fractional derivative appears more suitable to be treated by the Laplace transform technique in that it requires the knowledge of the initial values of the function and of its integer derivative of order $j = 1, 2, \dots, n-1$. The Caputo fractional derivative is a regularization in the time origin for the Reimann-Liouville fractional derivative by incorporating the relevant initial conditions.

Applying the Laplace transforms to the equations (4) and (10) and associated boundary conditions (5) to (9) a dimensionless pressure solution in Laplace space is obtained as

$$\bar{P}_{fD}(z) = \frac{K_v(z^{1/d_w})}{z[z^{1/d_w} K_{v+1}(z^{1/d_w}) - 2vK_v(z^{1/d_w})]} \quad (12)$$

$$\bar{P}_{\phi D} = [C_{\phi D} / z - C_{\phi D} / (z + \alpha_D^{-1})] \quad (13)$$

$$\text{and, } \bar{P}_{\phi D} = (C_{\phi D} / z) e^{-\alpha_D^2 z^2 / 4} \operatorname{erfc}(\alpha_D z / 2) \quad (14)$$

The general solution for wellbore pressure (\bar{P}_{wD}) in terms of \bar{P}_{fD} and $\bar{P}_{\phi D}$ in Laplace space is given as

$$\bar{P}_{wD}(z) = \frac{[z\bar{P}_{fD} + S][1 + C_D z^2 \bar{P}_{\phi D}]}{z[1 + C_{mD} z^2 + C_D z^{1/d_w} \{z\bar{P}_{fD} + S\}]} \quad (15)$$

Where C_D , S , $C_{\phi D}$, C_{mD} and d_w is wellbore storage constant, skin factor, changing wellbore storage parameter, momentum parameter and dynamic fractal dimension that effects the wellbore pressure measurements in the well. The equation (15) reduces to Park et al. [3], if $C_{\phi D} = 0$ and $C_{mD} = 0$. If z^{1/d_w} is replaced with $z^{1/2}$ and $C_{mD} = 0$, $C_{\phi D} = 0$, the equation reduces to the equation proposed by Chang and Yortsos [1]. The symbols used in the present paper are of standard type of petroleum industry and given in the reference [7]. The analytical inversion of equation (15) from Laplace space to real space is not possible. So, the numerical inversion technique of equation (15) from Laplace space to real space has been applied which is given by Stehfest algorithm [10].

In the present paper the new ratio of the form $\ln P_{wD} / \ln t_D$, (pressure derivative and the pressure data) are generated



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from the equation (15) for fractal reservoir. The well testing pressure derivative function defined by Bourdet et al. [11] is powerful mechanism for interpreting well test behavior (as in Figure C). It is, in fact, the most significant single development in the history of well test analysis. In this ratio curve we have two control points for matching the actual pressure data with the observed data i.e. the starting point and the last point of the curve. This gives the significant type curve match for the identification of the reservoir model to be used for automatic parameter estimation by using the regression analysis techniques. The curves generated are given in the figures 1-5. The pressure and its derivative match plot of observed data of well no. A by regression analysis is given in figure 6 for fracture of middle intensity [12].

Conclusions

In fractal reservoir, due to the slowdown of the diffusion, the bottom-hole pressure is less affected by the formation compared to that of Euclidean reservoir under the same wellbore storage. For fractal reservoir, the increase of the dynamic dimension d_w has the same effect of the increase of the wellbore storage effect. The results obtained in the present study may be useful for characterizing the fractality fractured reservoir and physical properties such as wellbore storage and skin have been compared with the Chang and Yortsos methods. The mathematical solution presented here is similar one recommended for pressure transient analysis of transient data from naturally fractured reservoir. The ratio curves obtained by pressure and its derivative are more useful to match the observed data on moving over the type curves presented here. The model is well suited for computer automated well test analysis from where permeability, fractal and spectral dimension, skin factor, wellbore storage and changing wellbore parameter and fault of impermeable/semi impermeable/fully permeable types can be predicted and the related parameter can be estimated. This method may provide a good way of matching reservoir response with simulated geology that is consistent with pressure dimensionless changing storage transient test data. The data of some of the well A has been matched and the parameters estimated are given in the figure 6.

Acknowledgements

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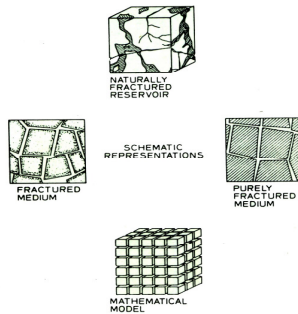


Fig. A SCHEMATIC REPRESENTATIONS OF A FRACTURED MEDIUM (after Warren & Root, Streltsova-Adams)

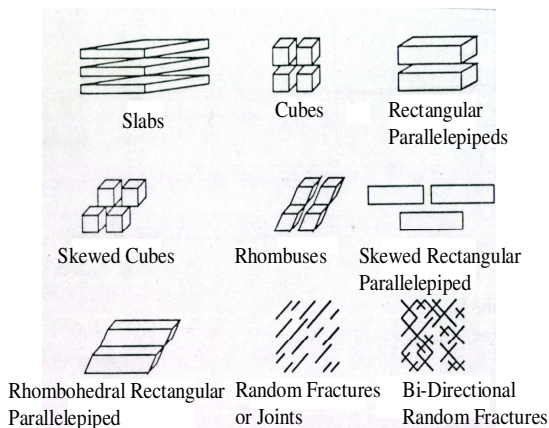


Fig. B Idealization of Typical Fracture Patterns seen in Nature

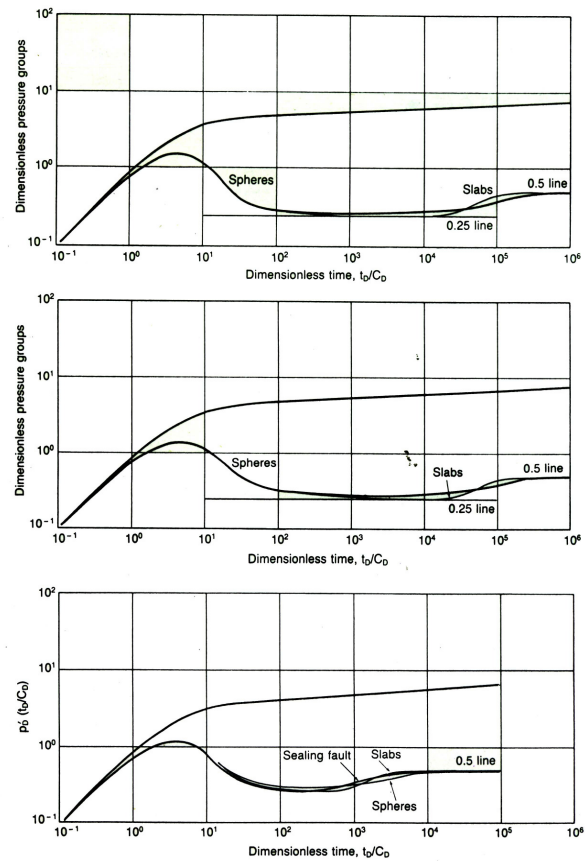


Figure: C Pressure Response of idealized fracture patterns



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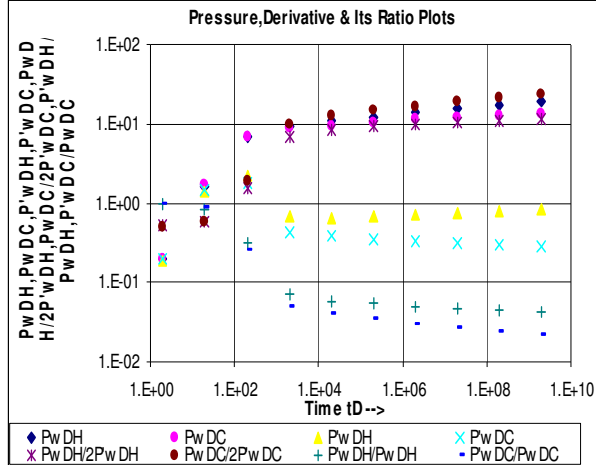


Figure 1: Pressure, derivative & its ratio curve for $C_D = 10$, $S = 5$, $d_w = 2.05$, $d_f = 2$, $C_{\Phi D} = 0$, $\alpha_D = 0$

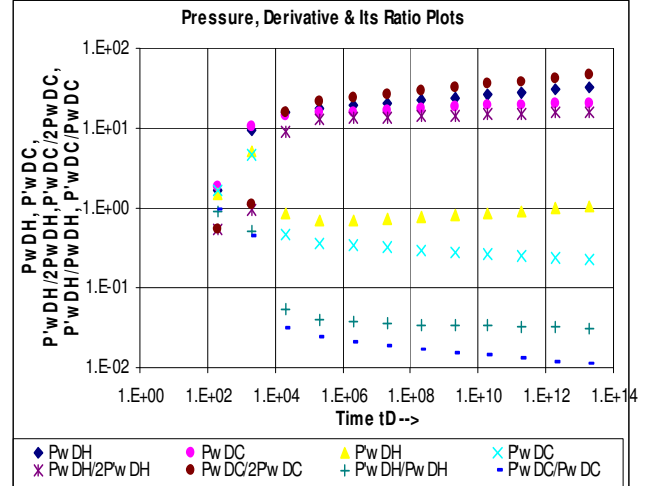


Figure 2: Pressure, derivative & its ratio curve for $C_D = 100$, $S = 10$, $d_w = 2.05$, $d_f = 2$, $C_{\Phi D} = -1.0$, $\alpha_D = 250$

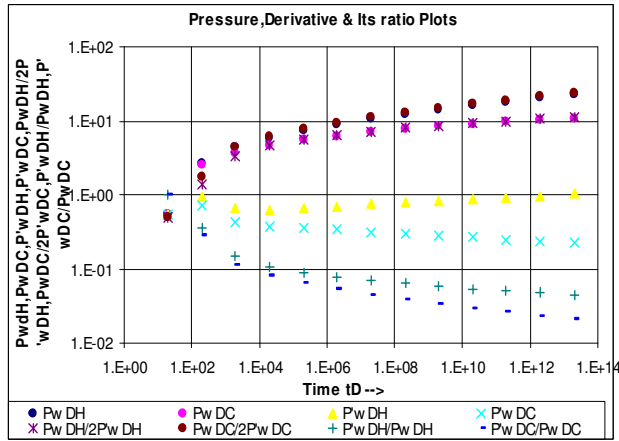


Figure 3: Pressure, derivative & its ratio curve for $C_D = 20$, $S = 0$, $d_w = 2.05$, $d_f = 2$, $C_{\Phi D} = -1.0$, $\alpha_D = 25$

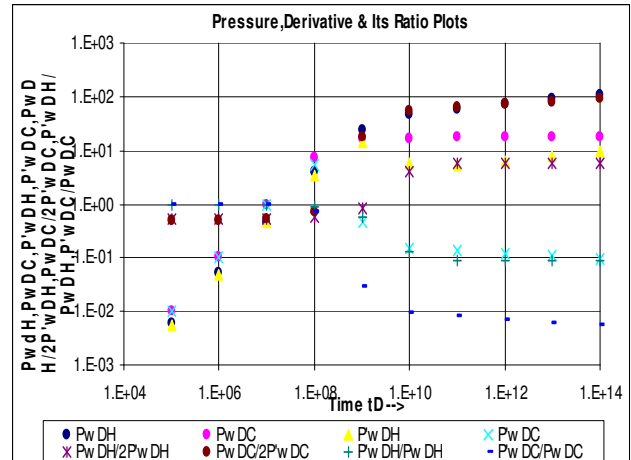


Figure 4: Pressure, derivative & its ratio curve for $C_D = 10 \times 10^6$, $S = 10$, $d_w = 2.21$, $d_f = 1.9$, $C_{\Phi D} = 0$, $\alpha_D = 0$



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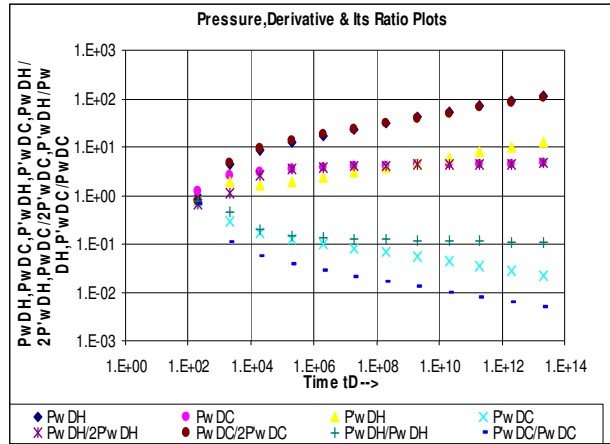


Figure 5: Pressure, derivative & its ratio curve for $C_D = 100$, $S = 0$, $d_w = 2.21$, $d_f = 1.95$, $C_{FD} = -1.0$, $\alpha_D = 25$

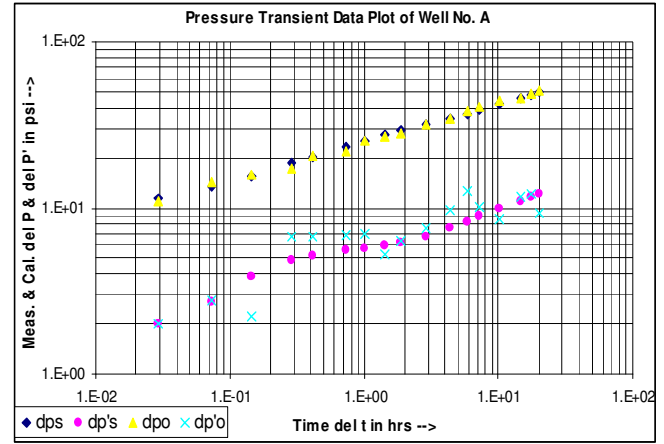


Figure 6: Match Pressure & derivative curve for $C_D = 0.005$, $S = -4.75$, $X_w = 582$ ft, $X_e = 1517$ ft, $d_w = 2$, $d_f = 2$, $\omega = 0.092$, $\lambda = 4.210^{-5}$, $k = 593$, md