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3D acoustic wave modeling with a time-space domain dispersion-relation-based Finite-difference scheme

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Summary

Spatial finite-difference (FD) stencils designed in the space domain are usually employed in wave equation modeling. In this paper, we adopt a spatial FD stencil which is devised in the time-space domain based on a dispersion relation to improve the accuracy of FD modeling for 3D acoustic wave equation. In addition, we employ a new hybrid absorbing boundary condition (ABC) to eliminate edge reflection due to finite computational domain. Dispersion analysis and modeling results demonstrate that the time-space domain dispersion-relation-based spatial FD stencils can indeed improve the modeling accuracy. The modeling accuracy can be improved further by using a truncated FD method. The new hybrid ABC can effectively absorb boundaries reflections. The modeling scheme presented in this paper is more accurate and efficient than the conventional one and can be used routinely.

Introduction

The finite-difference (FD) method is an important method for the numerical solution of partial differential equations and has been widely utilized in seismic modeling and migration (e.g., Kelly et. al, 1976; Dablain, 1986; Liu and Sen, 2009a). A high-order spatial FD is a common approach to increase modeling accuracy (e.g., Dablain, 1986; Fornberg, 1987; Etgen and O'Brien, 2007). A low-order FD algorithm uses a shorter operator but needs more grid points for discretization. A high-order FD algorithm uses a longer operator but needs fewer grid points. Since a conventional explicit high-order temporal FD is usually unstable in the wave equation modeling, spatial derivatives are used to replace high-order temporal derivatives (e.g., Dablain, 1986) to increase the accuracy of temporal derivatives with additional computational cost.

Generally, most FD methods determine the FD stencils for spatial derivatives only in the space domain. However, the seismic wave propagation calculation is done both in space and time domains. To improve the accuracy of conventional FD methods, a new time-space domain FD method was proposed to derive the spatial FD coefficients in the joint time-space domain (Finkelstein and Kastner, 2007). The key idea of the method is that the dispersion relation is completely satisfied at designated frequencies and thus the spatial FD coefficients are frequency dependent. This method was developed further for 1D, 2D and 3D acoustic wave modeling using a plane wave theory and the Taylor series expansion (Liu and Sen, 2007b). These new spatial FD coefficients, dependent on the Courant number and space point number, are frequency independent though they lead to a frequency dependent numerical solution. In this paper, we report on the numerical results in 3D using FD that replaces the conventional 3D spatial FD coefficients with these new coefficients to improve the modeling accuracy without increasing the calculation amount. A truncated FD method is also used to enhance the modeling accuracy further. A hybrid absorbing boundary condition (ABC) is also developed for 3D acoustic modeling.

Conventional finite-difference scheme for the 3D acoustic wave equation

We start with the 3D acoustic wave equation given by

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2 p}{\partial t^2}, \qquad (1)$$

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Where, $p = p(x, y, z, t)$ is a scalar wave field, and v is velocity.

The following 2nd-order FD is usually used for the time derivatives,

$$\frac{\partial^2 p}{\partial t^2} \approx \frac{\delta^2 p}{\delta t^2} = \frac{1}{\tau^2}\left[-2p_{0,0,0}^0 + \left(p_{0,0,0}^{-1} + p_{0,0,0}^1\right)\right], \tag{2}$$

where,

$$p_{m,l,j}^n = p(x+mh, y+lh, z+jh, t+n\tau), \tag{3}$$

τ is time step, h is grid size. Generally, the modeling accuracy is improved by high-order FD on the space derivatives given by

$$\frac{\partial^2 p}{\partial x^2} \approx \frac{\delta^2 p}{\delta x^2} = \frac{1}{h^2}\left[a_0 p_{0,0,0}^0 + \sum_{m=1}^{M} a_m\left(p_{-m,0,0}^0 + p_{m,0,0}^0\right)\right], \tag{4a}$$

$$\frac{\partial^2 p}{\partial y^2} \approx \frac{\delta^2 p}{\delta y^2} = \frac{1}{h^2}\left[a_0 p_{0,0,0}^0 + \sum_{m=1}^{M} a_m\left(p_{0,-m,0}^0 + p_{0,m,0}^0\right)\right], \tag{4b}$$

$$\frac{\partial^2 p}{\partial z^2} \approx \frac{\delta^2 p}{\delta z^2} = \frac{1}{h^2}\left[a_0 p_{0,0,0}^0 + \sum_{m=1}^{M} a_m\left(p_{0,0,-m}^0 + p_{0,0,m}^0\right)\right]. \tag{4c}$$

A Taylor series expansion is generally used to determine the FD coefficients (Fornberg, 1987; Dablain, 1986). The FD coefficients in Equations (4a), (4b) and (4c) are as follows (Liu and Sen, 2009a)

$$a_m = \frac{(-1)^{m+1}}{m^2} \prod_{1<n\le M, n\ne m} \left|\frac{n^2}{n^2 - m^2}\right| \quad (m = 1, 2, ..., M), \tag{5a}$$

$$a_0 = -2\sum_{m=1}^{M} a_m . \tag{5b}$$

It can be proved that when a 2*M*-order FD space stencil and a 2nd-order FD time stencil are used to solve the 3D acoustic wave equation, numerical modeling has 2nd-order accuracy. It is noteworthy that increasing M may help reduce the magnitude of the conventional FD error without increasing the accuracy order.

Time-space domain dispersion-relation-based finite-difference scheme for the 3D acoustic wave equation

For the time-space domain dispersion-relation-based FD scheme, spatial FD coefficients can be derived by using the following steps (Liu and Sen, 2009b). First, derive dispersion relation of FD modeling by using plane wave theory and Equations (1)-(4). Second, apply the Taylor series expansion for trigonometric functions in the dispersion relation equation. Then, compare coefficients of power series and obtain the following equations

$$\sum_{m=1}^{M} m^{2j}\left(\cos^{2j}\theta\cos^{2j}\phi + \cos^{2j}\theta\sin^{2j}\phi + \sin^{2j}\theta\right)a_m = r^{2j-2}$$

$$(j = 1, 2, ..., M), \tag{6}$$

where, $r = v\tau/h$, θ is the plane wave propagation angle measured from the horizontal plane perpendicular to z axis, ϕ is the azimuth of the plane wave. Note that these equations indicate that the coefficients a_m are a function of θ and ϕ. We solve Equation (6) to obtain a_m by using an optimal direction with $\theta = 0$ and $\phi = \pi/8$ (Liu and Sen, 2009b), which makes FD modeling attain the highest (2*M*)th-order accuracy along 48 directions.

Absorbing boundary condition

Here, we adopt the new hybrid scheme developed by Liu and Sen (2009c) to absorb reflections from the model boundaries in numerical solutions of wave equations (WEs). This scheme divides the computational domain into boundary, transition and inner areas. The wavefields within the inner and the boundary areas are computed by the WE and the one-way wave equation (OWWE) respectively. The wavefields within the transition area are determined by a weighted combination of the wavefields computed by WE and the OWWE to obtain a smooth variation from the inner area to the boundary via the transition zone.

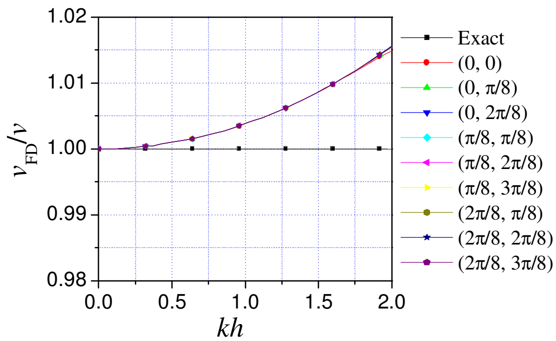
We develop this hybrid ABC for 3D acoustic wave modeling. For 3D modeling, boundaries include six sides, twelve edges and eight corners. Second-order OWWEs are adopted for these sides, and first-order OWWEs for these edges and corners (Clayton and Engquist, 1977). Ten grid points are used to define the transition area to absorb boundary reflections.

Dispersion analysis

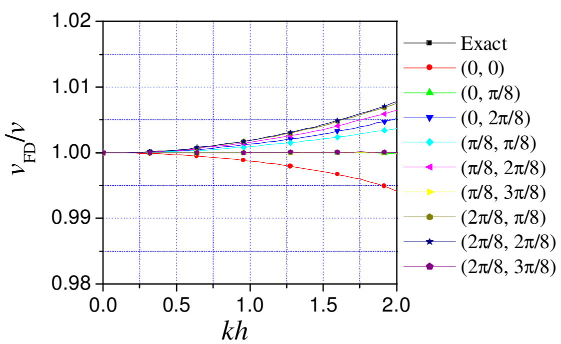
The dispersion of FD for 3D modeling is described by the following expression (Liu and Sen, 2009b),

$$\delta = \frac{v_{\mathrm{FD}}}{v} = \frac{2}{rkh}\sin^{-1}\sqrt{r^2\sum_{m=1}^{M} a_m \begin{pmatrix} \sin^2(mkh\sin\theta/2) \\ +\sin^2(mkh\cos\theta\sin\phi/2) \\ +\sin^2(mkh\cos\theta\cos\phi/2) \end{pmatrix}}\ , \tag{7}$$

Where, k is the wavenumber. If δ equals 1, there is no dispersion. If δ is far from 1, a large dispersion will occur. Since kh is equal to π at the Nyquist frequency, when calculating δ, kh only ranges from 0 to π. Figure 1 illustrates the variation of dispersion parameter $\delta(\phi,\theta)$ with kh along nine directions, which demonstrates that the accuracy of the time-space method is greater than that of the conventional method.



(a) Conventional method



(b) Time-space method

Figure 1: Plot of dispersion curves of the conventional and the time-space FD methods for 3D acoustic wave equation modeling. $M = 10$, $v = 3000$m/s, $\tau = 0.001$s, $h = 10$m.

Numerical modeling examples

First, both the conventional and the time-space FD methods are used to simulate 3D acoustic wave propagation in a homogeneous acoustic medium under the same discretization. The model and simulation parameters are listed in the caption of Figure 2. Computed snapshots respectively by the conventional and the time-space FD methods are shown in Figures 2(a) and 2(b). Comparing these two figures, we can see that the time-space method maintains the waveform better than the conventional method and thus has greater precision.

To improve the modeling accuracy further, we introduce the truncated FD method into the time-space FD method. It is known that with the increase of the number of grid points involved in FD discretization, the accuracy increases but the computational cost also increases. Liu and Sen (2009d) found that there exist some very small coefficients for high-order FD coefficients and with the increase of order the number of these small coefficients increases but their values decrease sharply. They also demonstrated that omitting these small coefficients can maintain approximately the same level of accuracy of FD but reduce computational cost significantly. Figure 2(c) shows snapshots by the truncated time-space method, it follows that the dispersion decreases further compared with Figure 2(b).

Next, we adopt the truncated time-space FD method to perform numerical modeling. Figure 3 displays the seismograms computed for a horizontally layered acoustic model with the Clayton-Engquist ABC (Clayton and Engquist, 1977) and the new hybrid ABC. The model and simulation parameters are given in the figure caption. The figure suggests that the boundary reflections are still strong compared with the reflections from the true reflectors for the Clayton-Engquist ABC. The new hybrid ABC achieves nearly perfect absorption.

Conclusions

We have developed a time-space domain dispersion-relation-based FD scheme combined with the truncated FD method and the hybrid ABC for 3D acoustic wave equation modeling. Dispersion analysis and numerical modeling results demonstrate that this scheme has greater accuracy and can effectively suppress dispersion and boundary reflections.

Acknowledgments

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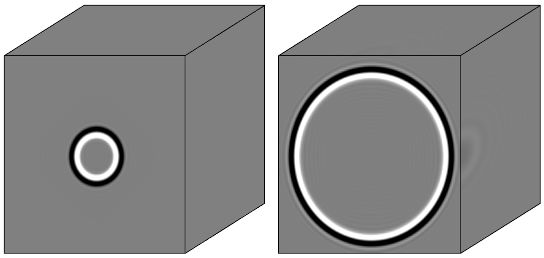
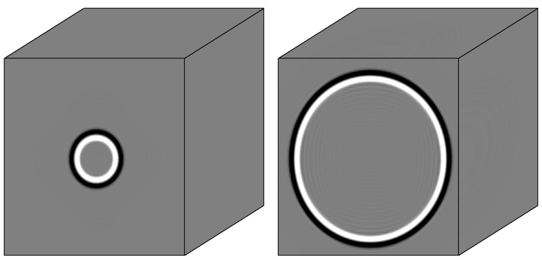
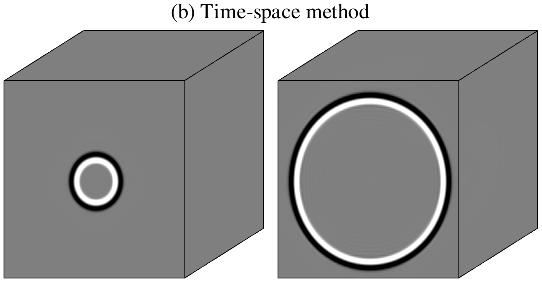


(a) Conventional method

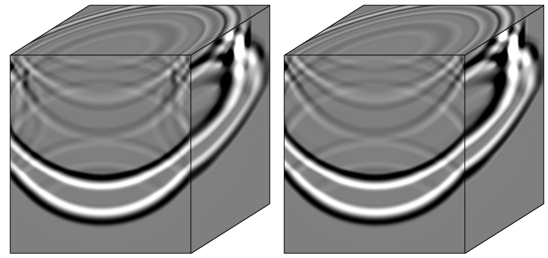


(b) Time-space method

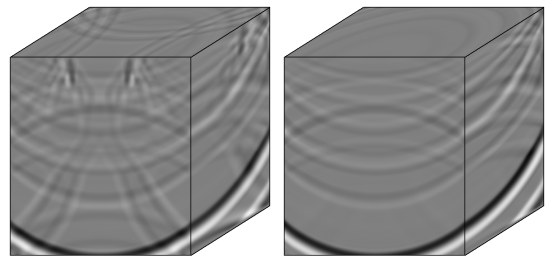


(c) Truncated time-space method

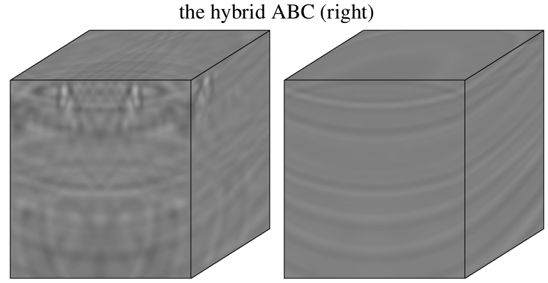
Figure 2: Snapshots computed by finite-difference modeling for a 3D homogeneous acoustic model respectively with conventional, time-space and truncated time-space FD methods. Each figure includes 2 panels with 2 snapshots at 50ms and 150ms from left to right. The model velocity is 3000m/s, grid size is 10m×10m×10m, grid dimensions are 100×100×100, grid point coordinates range from (0,0,0) to (990m,990m,990m), time step is 1ms, M=10. The same discretization is involved in all the three methods, but their spatial FD coefficients are different. A source pulse of 50Hz sine function with one period length is located at the center of the model. The free surface condition and ABC are not included here. Top, front and right surfaces of snapshots are recorded at z=0m, y=500m and x=990m respectively.



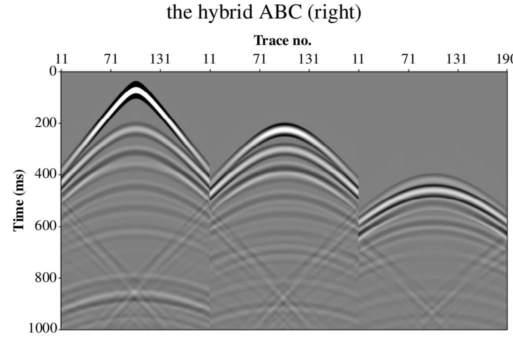
(a) Snapshots at t=500ms by the Clayton-Engquist ABC (left) and the hybrid ABC (right)



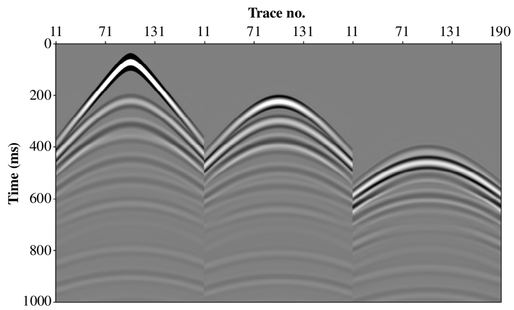
(b) Snapshots at t=700ms by the Clayton-Engquist ABC (left) and the hybrid ABC (right)



(c) Snapshots at t=1000ms by the Clayton-Engquist ABC (left) and the hybrid ABC (right)



(d) Seismograms with the Clayton-Engquist ABC



(e) Seismograms with the hybrid ABC

Figure 3: Snapshots and seismograms computed by truncated time-space FD modeling for 3D horizontally layered acoustic model respectively with the Clayton-Engquist ABC and the hybrid ABC. Each figure of seismograms includes 3 receiver lines. The model has 6 layers, whose velocities are velocity are 2500m/s, 3000m/s, 3500m/s, 3000m/s, 3600m/s and 4000m/s from shallow to deep, 5 interfaces depth are 300m, 500m, 800m, 1200m and 1400m. Grid size is 10m×10m×10m, grid dimensions are 200×200×200, grid coordinates range from (0m,0m,0m) to (1990m,1990m,1990m), time step is 1ms. M=10. A source pulse of 20Hz sine function with one period length is located (990m, 990m, 100m). Receivers are located on the surface, and 3 receiver lines shown in this figure are located at y=990m, 1490m and 1990m from left to right. Trace no. of seismogram for each receiver line ranges from 1 to 200. Traces whose no. varies from 11 to 190 are shown since 10 grids of width are used for the hybrid ABC. The free surface condition is included here. Top, front and right surfaces of snapshots are recorded at z=0m, y=1000m and x=1990m respectively.

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