Introduction

Stochastic simulation was introduced by Matheron (1973) and Journel (1974) to correct for the smoothing effects and other artifacts of kriging, allowing the reproduction of spatial variance predicted by the variogram model. Stochastic simulation stepped away from the variogram and kriging based core for the first time with the Boolean object-based algorithms, introduced by Stoyan, Kendall and Mecke (1987), and Haldorsen and Damsleth (1990), in an attempt to reproduce geological features, like channels and fractures. Object based algorithms have difficulties honoring all available well data because they require iterative “trial and error” modification. Initially Srivastava (1992), and later Caers (1998) and Strebelle (2000), proposed the idea of borrowing conditional probabilities directly from a training image, allowing the use of higher order or multiple point statistics to reproduce geological structures and patterns.

Training images and multiple point statistics methods remained largely unpopular until developments in computational capability and multiple point scanning techniques took place. Currently, the basic components that characterize the application of multiple point geostatistics remain the major focus of continuous research and development efforts. These components include, among others, the generation or acquisition of numerical spatial representations to be used as training images; the optimal definition of scanning templates to capture the proper conditional information from the training image; and development of computational schemes to scan, store and reproduce complex spatial features.

Scanning template is an interface for retrieving mp statistics from training images. Despite the great impact of the template on the reproduced patterns it has not received considerable attention in previously developed mp statistics algorithms. The reliability of the final reproduced patterns is highly sensitive to the template shape and size. A new algorithm for shaping the optimal template (give better pattern reproduction results) is integrated with the current algorithm. The optimal template selection procedure defines the optimal template’s shape and size consistent with observed objects in the training images.

Quantifying and scanning the training images using the template as an interface and keeping track and storing different observed patterns in an efficient way has received considerable research focus in the recent years. The k-d tree algorithm by Bently (1975) and the optimized version by Jerome H. Friedman and Jon Louis Bently (1976) is a generalization of the simple binary tree used for sorting and searching data. The k-d tree is a binary tree in which each node represents a subfile of the records in the file and a partitioning of that subfile. Each observed pattern while scanning the training image is expressed in the form of the long string where each nine digits of the string will appear in the different file. The first arrays which contain first nine digit of each string are like the first set of sub nodes of the main root. The first nine digits of many different patterns might be similar but the second sub arrays may not be, and if first and second sub arrays are similar, the third one may not be and so on. This method of comparison will help to calculate the frequency of each distinct pattern and build the mp histogram.

Reproducing patterns in modeling underground oil reservoirs by incorporating mp statistics or patterns extracted from training images is one of the main research areas in stochastic reservoir modeling. Pierre Goovaerts and Andre G. Journel (1996) used simulated annealing to honor mp statistics borrowed from a training image of categorical facies data. Strebele (2000) developed a MPS algorithm for pattern reproduction. That algorithm sequentially simulates one node at a time conditioned to the hard data. Conditional probabilities at each step are built based on the frequencies of different patterns extracted from training images. MPS algorithms, initially, were developed to work within stationary framework. Strebele, Payarzayan and Caers (2002) and Strebele and Zhang (2004) modified the MPS such that it accounts for auxiliary constraints.
In contrast to previous approaches for multiple points simulation where the pattern is simulated one node at a time, this paper will introduce a new MPS algorithm which simulates multiple point patterns conditioned to multiple point data events. The patterns are thus grown from well or already simulated data locations. These seed nodes are called simulatable nodes.

**Multiple Point Pattern Simulation**

In this work, the mp simulation framework is extended to permit the inference and simulation of a spatial pattern (multiple points) conditioned to the pattern (multiple point) exhibited by the data in the neighborhood. This subtle and yet important shift in mp paradigm results in a much more efficient, growth-based methodology to simulate models exhibiting complex structures. Training Image, Optimal Template, Pattern Statistics and Pattern Reproduction are the main components of this new multiple point approach.

1) **Training Image**

This is a digital representation of the spatial law or explicit non-conditional conceptual description of the geological structures and patterns to be reproduced in the model. Training images require prior information about the depositional environment and these images can be obtained from outcrops and photographs. Object based algorithms become handy for building training images. These images are good for depicting reservoirs with distinct geological features.

2) **Template Selection**

The spatial law which will be implemented in the pattern reproduction algorithm is fully characterized by the template. The size and geometry of a suitable scanning template should meet certain criteria to identify the most relevant geological features. A fast and robust algorithm to derive optimal spatial templates is presented that is based on a semi-automated procedure. Given a user specified template grid, the algorithm calculates the covariance between any node in that grid and the central node, by scanning the training image with that particular two-point template. These covariances are then sorted and the nodes with the highest covariance values are retained as part of the optimal template. Consequently, the optimal template is identified and configured to better capture structures and patterns from the training image.

3) **Pattern Statistics**

Multiple point statistics in the form of the frequency of observed data configuration, patterns, are recorded. The combination of patterns in a reasonable size template can be so large as to make storage of the statistics in a conventional integer format within a program impractical. In order to overcome that problem, a tree concept was devised that helps keep track of pattern occurrences in an efficient manner. The methodology is to store each nine digits of the pattern in a different array then compare patterns by comparing the first nine digits, if they are equal then comparing the second nine digits and so on. Further efficiencies to the task of pattern inference are possible by implementing the procedure in a distributed computing environment.

4) **Pattern Reproduction**

A pixel-based non-iterative process to perform conditional simulation of complex geological structures was proposed by Srivastava (1992), explored further by Caers (1998) and Strebelle (2000). In all those approaches, the simulated image is generated one node at a time. In contrast, the proposed approach; instead of building the simulated image node by node, simulates an entire pattern in the nodes of the templates conditioned to the pattern exhibited by the data on the template. The algorithm is therefore growth based, with the patterns grown from seed nodes where data is available. At any step in the simulation algorithm, a list of simulatable nodes is created. These simulatable nodes are chosen such that when the optimal spatial template is centered at that location, there is at least one conditioning data located on the template. A node is selected at random from this list of simulatable nodes, and the simulation is performed. The probability corresponding to patterns are obtained by a single application of Bayes’ rule:

\[
P( A | B ) = \frac{ P(A, B) }{ P(B) }
\]

Where \( A \) is the simulation pattern and \( B \) is the conditioning pattern, \( P(A,B) \) is the joint probability of the simulation and conditioning pattern obtained from the multiple point histogram and \( P(B) \) is the prior probability of the conditioning patterns, also obtained from the original histogram.

**Non-Stationarity and Proportional Maps**

MPS algorithms were initially developed to work within a stationary framework. The MPS algorithm developed in the previous section has been modified so that it accounts for auxiliary constraints. The non-stationary is honored by perturbed mp statistics within non-stationarity regions in order to simulate models that exhibit different characteristics in different regions.

**Real Field Example**
The algorithm has also been tested using a real field data set. The field is a channel system with more than 140 wells, most of which have well logs. The reservoir was modeled using 5 main layers and 25 sub-layers. This was done so as to be able to match the vertical proportions accurately and at the same time retain realistic shapes for the channel crosssections. As is shown in Figure 1 (net sand contour map for the “Morow” production zone), there are three channel fairways in the field. Using the given contour maps, the vertical proportional maps (Figure 2) are built that depicts the variations in channel proportion over the 5 layers. These maps serve to constrain higher channel density within the channel fairways. Figure 1 also shows the simulation area along with conditional data locations. These conditional data (from 66 wells for all 25 sub-layers) are inputs to the algorithm and are integrated (honored) in the final simulated image.

**Reservoir Training Image**

The reservoir is considered to be made up of channel and non-channel facies. A 3D training image is generated using Fluvsim which is an object based stochastic simulator for channel simulation.

**Results**

The conditional hard data for 25 sub-layers, proportional maps and a training image are input to the algorithm. The optimal template computed by the algorithm is then used to build the mp histogram and finally grow the pattern using mp histogram.

The 3D simulated cube is shown in Figure 3. These images have been obtained using e60Vision’s proprietary visualization software. The images depict the connectivity of the channel network and the correct trajectory of the channel fairways.
**Pattern Growth**

The pattern reproduction part of the whole framework is unique; the simulated patterns are grown from well locations or from already simulated data locations and eventually the whole reservoir domain is covered. These seed nodes are called simulatable nodes. The growing nature of the algorithm disturbs the simulatable nodes list. Thus, the simulatable nodes list after simulating each pattern either expands or shrinks. Figure 4 depicts the pattern growth at various stages of the simulation.

**Discussion and Conclusion**

A multiple point framework for pattern reproduction (reservoir modeling) is presented in this paper. Within the developed framework, static data (well data, areal and vertical proportions and conceptual geological information) can be integrated into the reservoir models. The approach is centered on the development of a new multiple point simulation algorithm that utilizes an optimal spatial template and a unique growth-based algorithm for pattern reproduction. The algorithm has been tested on a real field data and the reproduced patterns are consistent with geological information. The models display the geological connectivity that could potentially be important for making accurate flow predictions.

Integrating production history data, while preserving geological consistency, is an important research challenge. The future work is to expand the mp simulation algorithm to integrate dynamic data within the available growth-based approach; the final reproduced patterns then would honor static and dynamic data.

**References**


